Lie Algebras and Representation Theory
Algèbres de Lie et théorie des représentations
(Org: **Nicolas Guay** (Alberta) and/et **Michael Lau** (Windsor))

PUNITA BATRA, Harish Chandra Research Institute, Chhatnag Road, Jhunsi, Allahabad, India
Classification of Integrable representations for twisted toroidal Lie algebras

I will talk about the irreducible, integrable modules for twisted toroidal Lie algebras with finite dimensional weight spaces. These modules turn out to be modules of direct sums of finitely many copies of affine Kac–Moody Lie algebras.

ELAINE BELTAOS, Grant MacEwan University, P.O. Box 1796, Edmonton, AB T5J 2P2
Fixed Point Factorization and Applications

The affine Kac–Moody algebras give rise to rational conformal field theories (RCFTs), which are two-dimensional quantum field theories that are symmetric under conformal transformations, and also satisfy a finiteness condition. A key ingredient of an RCFT is its modular data—two matrices S, T that generate a representation of $\mathrm{SL}_2(\mathbb{Z})$. Fixed point factorization is a technical tool that dramatically simplifies the S-matrix at entries involving 'fixed points'. Fixed points often present complications, and fixed point factorization provides a way to handle these. In this talk, we will discuss fixed point factorization and an interesting application of it in mathematical physics.

This work was part of the speaker's doctoral thesis under the supervision of Professor Terry Gannon at the University of Alberta.

GEORGIA BENKART, University of Wisconsin–Madison, 480 Lincoln Dr., Madison, WI 53706, USA
Motzkin Algebras

This talk will feature a certain algebra of diagrams called the Motzkin algebra, which was introduced in our recent joint work with T. Halverson. These algebras have beautiful algebraic and combinatorial properties and are are related to Temperley–Lieb algebras and to the representation theory of quantum enveloping algebra $U_q(\mathrm{sl}_2)$.

CHRIS BRAV, University of Toronto
Smaller resolutions of Pfaffian varieties

A rank variety is the locus of matrices of some kind having rank less than or equal to some fixed non-negative integer. Typically, rank varieties are singular and have standard resolutions given by total spaces of vector bundles over Grassmannians. When such resolutions are small, the cohomology of the resolution is isomorphic to the intersection cohomology of the rank variety.

In the case of a skew-symmetric rank variety, the standard resolution fails to be small and we show how to replace it with a smaller non-commutative resolution whose Grothendieck group is isomorphic to the intersection cohomology of the rank variety.

XUEQING CHEN, University of Wisconsin–Whitewater
Algebraic Structures over Derived Categories and Triangulated Categories

Through the Ringel–Hall algebra approach, one can construct Kac–Moody Lie algebras and some elliptic Lie algebras from the derived categories of some finite dimensional associative algebras. In this talk, we start by recalling Peng–Xiao's work on the construction of Kac–Moody algebras from the derived categories of hereditary algebras, Lin–Peng's work on the construction of some elliptic algebras from the derived categories of some tubular algebras, and Toën's work on the construction of derived Hall algebras over differential graded category under some finiteness conditions. Then we discuss some generalizations of the above results and prove an analogue of Toën's formula which is used to define derived Hall algebras for odd-periodic triangulated categories. As an example, the Hall algebra over the 3-periodic orbit triangulated category of a hereditary abelian category will be described.

This talk is based on a joint work with F. Xu.

GERALD CLIFF, University of Alberta, Edmonton, AB T6G 2G1

Explicit bases for representations of Lie algebras and superalgebras

We will discuss explicit bases of modules for semi-simple Lie algebras and for general linear Lie superalgebras. We will also discuss explicit bases for coordinate rings of reductive algebraic groups.

IVAN DIMITROV, Queen's University, Kingston, Ontario K7L 3N6

Weight modules over classical infinite-dimensional Lie algebras

Simple weight modules with finite-dimensional weight spaces over reductive Lie algebras were classified by Fernando and Mathieu. In this talk I will discuss analogs of this classification for the classical infinite-dimensional Lie algebras A_∞, B_∞, C_∞, and D_∞. There are several new features that distinguish the infinite-dimensional case from the finite-dimensional one. For example, the analog of the Fernando–Futorny parabolic induction theorem fails to hold. Nevertheless, a complete classification can be obtained for modules satisfying mild additional conditions. A prominent role in this classification is played by the so called pointed weight modules, i.e., modules with one-dimensional weight spaces only. The description of all simple pointed modules is derived from the results of Benkart, Britten, and Lemire about pointed modules over simple finite-dimensional Lie algebras.

JOEL KAMNITZER, University of Toronto

Categorical Lie algebra actions and braid groups

We will discuss the notion of categorical Lie algebra actions, as introduced by Rouquier and Khovanov–Lauda. In particular, we will give examples of categorical Lie algebra actions on derived categories of coherent sheaves. We will show that such categorical Lie algebra actions lead to actions of braid groups.

RINAT KEDEM, University of Illinois

Integrable cluster algebras

I will give an overview of recent results with Ph. Di Francesco about the solutions of cluster algebras associated with Q-systems and T-systems, as well as related discrete integrable systems. Solutions admit a description as path partition functions, and this description is particularly well-suited to the generalization to the non-commutative or quantum case.

JOCHEN KUTTLER, University of Alberta, Edmonton, Alberta

Schubert varieties in the affine Grassmannian

I will present some results regarding singularities of Schubert varieties in the affine Grassmannian (i.e., the quotient $\mathrm{SL}_n(F)/\mathrm{SL}_n(A)$ where F is the field of Laurent series and A is the ring of formal power series).

This is joint work with Lakshmibai.

SILVIA MONTARANI, University of Toronto, Department of Mathematics, 40 St. George St., Room 6290, Toronto, ON M5S 2E4

Representations of Gan–Ginzburg algebras and quiver-related differential operators

Gan–Ginzburg algebras are one-parameter deformations of the wreath product of a symmetric group with a deformed preprojective algebra for a quiver Q. When Q is extended Dynkin, these algebras are related to the symplectic reflection algebras of Etingof and Ginzburg, and when Q is star-shaped, but not finite Dynkin, they contain a subalgebra isomorphic to a Generalized Double Affine Hecke Algebra (GDAHA). In this talk, we will explain how to construct representations of Gan–Ginzburg algebras starting from modules over the algebra of differential operators on a space of representations of the quiver Q. Time allowing, we will present a Lie theoretic construction of representations for GDAHAs, and show how some of these representations can be obtained by restriction from the representations of Gan–Ginzburg algebras we introduced.

ERHARD NEHER, University of Ottawa, 585 King Edward, Ottawa, Ontario K1N 6N5
Finite-dimensional irreducible representations of equivariant map algebras

We consider an affine algebraic variety X, a finite-dimensional simple Lie algebra L and a finite group G acting on both X and L by automorphisms. The space of G-equivariant regular maps from X to L is a Lie algebra under pointwise multiplication, called an equivariant map algebra. Examples of equivariant map algebras are (twisted or untwisted) multiloop algebras, current algebras, n-point Lie algebras, and the Onsager (Lie) algebra.

In this talk I will present a classification of finite-dimensional irreducible representations of equivariant map algebras: They are (almost) all evaluation representations. This result recovers the previously known classifications, for example for the multiloop, current and Onsager algebras. In addition, we can easily derive the precise structure of the finite-dimensional irreducible representations in previously unknown cases. Some examples will be presented.

The talk is based on joint work with Alistair Savage and Prasad Senesi.

ALISTAIR SAVAGE, University of Ottawa
Quiver grassmannians, quiver varieties and the preprojective algebra

Quivers play an important role in the representation theory of algebras, with a key ingredient being the path algebra and the preprojective algebra. Quiver grassmannians are varieties of submodules of a fixed module of the path or preprojective algebra. We show that the quiver grassmannians corresponding to submodules of certain injective modules are homeomorphic to the lagrangian quiver varieties of Nakajima which have been well studied in the context of geometric representation theory. We then refine this result by finding quiver grassmannians which are homeomorphic to Demazure quiver varieties and others which are homeomorphic to the graded/cyclic quiver varieties defined by Nakajima. The Demazure quiver grassmannians allow us to describe injective objects in the category of locally nilpotent modules of the preprojective algebra. We conclude by relating our construction to a similar one of Lusztig using projectives in place of injectives.

This is joint work with Peter Tingley.

JIE SUN, University of Ottawa
Universal central extensions of infinite dimensional Lie algebras

Central extensions play an important role in the theory of infinite dimensional Lie algebras. They allow one to construct bigger Lie algebras in a controlled way, which often have a more interesting representation theory than the original Lie algebra. A prime example is the construction of the (derived algebra of the) affine Kac–Moody algebra as the universal central extension of a twisted or untwisted loop algebra.

In this talk I will describe various constructions of (universal) central extensions. Special emphasis will be given to multiloop algebras and, more generally, Lie algebras that arise as twisted forms of (generalized) current algebras.

KAIMING ZHAO, Wifrid Laurier University
Supports of weight modules over Witt algebras

So far there is no classification for simple weight modules with finite dimensional weight spaces over Witt algebras W_n (or W_n^+). In this talk, we will explicitly describe supports of such modules over W_n. We also give some descriptions on the support of an arbitrary simple weight module over a $\mathbb{Z}^n$-graded Lie algebra $\mathfrak{g}$ having a root space decomposition $\bigoplus_{\alpha\in\mathbb{Z}^n}\mathfrak{g}_\alpha$ with respect to the abelian subalgebra $\mathfrak{g}_0$, with the property $[\mathfrak{g}_\alpha,\mathfrak{g}_\beta]=\mathfrak{g}_{\alpha+\beta}$ for all $\alpha,\beta\in\mathbb{Z}^n$, $\alpha\neq\beta$ (this class contains the algebras W_n).

This talk is based on a joint work (arXiv:0906.0947) with V. Marzuchuk.