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On tangential and normal criteria for convexity of sets in Banach spaces

We present criteria for convexity of closed sets in Banach spaces in terms of Clarke and Bouligand tangent cones. In the case of uniformly convex Banach spaces we also state such criteria in terms of proximal normal cones.

The main result is:

Theorem 1 *For closed subsets S of Banach space X the following relations are equivalent:*

(a) S *is convex;*

(b) $S \subset x + T_S^C(x)$ *for any* $x \in S$*;*

(c) $S \subset x + T_S^B(x)$ *for any* $x \in S$*. In the case of uniformly convex Banach space* X*, the previous relations are also equivalent to the next one;*

(d) $S \subset x + (N_S^P)^*(x)$ *for any* $x \in S$*.*

Here $T_S^C(x)$ is Clarke tangent cone to the set S at x, $T_S^B(x)$ is Bouligand tangent cone to the set S at x and $(N_S^P)^*(x)$ denotes the dual to a proximal normal cone to S at x.

As an example of application of criteria for convexity we derive sufficient conditions for convexity of images $F(C_\gamma)$ of a set

$$C_\gamma := \{x \mid V_i(x) \le \gamma_i, i = 1, \dots, m\}$$

under nonlinear mapping $F\colon X \to Y$.

This is a joint work with Francis Clarke of Institut Camille Jardine, Université Claude Bernard, Lyon, France.