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Schwarzian derivatives of convex mappings

For a function f analytic and locally univalent in the unit disk $\mathbb{D}$, the Schwarzian derivative is $Sf = (f''/f')' - \frac{1}{2}(f''/f')^2$ and the Schwarzian norm is $\|Sf\| = \sup_{z\in\mathbb{D}}(1-|z|^2)^2|Sf(z)|$. In 1949, Nehari proved that $\|Sf\| \le 2$ implies that f is univalent in $\mathbb{D}$. In 1976 he showed conversely that $\|Sf\| \le 2$ if f maps the disk univalently onto a convex region. We give a simple proof based only on the Schwarz lemma.

Nehari also claimed to show that $\|Sf\| < 2$ for bounded convex mappings. We verify this by proving more generally that the image of a convex mapping with $\|Sf\| = 2$ cannot be a quasidisk.

Joint work with Martin Chuaqui and Brad Osgood.