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Signatures of Hermitian forms and unitary representations

Suppose G is compact Lie group. The representations of G—possible ways of realizing G as group of matrices—provide a powerful way to organize the investigation of a wide variety of problems involving symmetry under G. For example, if G acts by isometries on a Riemannian manifold, each eigenspace of the Laplace operator is a representation of G. Knowing the possible dimensions of representations can therefore tell you about possible multiplicities of Laplacian eigenvalues.

When G is noncompact, there may be no realizations of G using finite matrices, and those involving arbitrary infinite matrices are too general to be useful. Stone, von Neumann, Wigner, and Gelfand realized in the 1930s that unitary operators on Hilbert spaces provided a happy medium: that any group could be realized by unitary operators, but that the possible realizations could still be controlled in interesting examples.

Gelfand's "unitary dual problem" asks for a list of all the realizations of a given group G as unitary operators. Work of Harish-Chandra, Langlands, and Knapp–Zuckerman before 1980 produced a slightly longer list: all realizations of G as linear operators preserving a possibly indefinite Hermitian form. I will describe a notion of "signatures" for such infinite-dimensional forms, and recent work of Jeff Adams' research group "Atlas of Lie groups and representations" on an algorithm for calculating signatures. This algorithm identifies unitary representations among Hermitian ones, and so promises to resolve the unitary dual problem.