**Contributed Papers**
**Communications libres**
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**MARIA BARBERO**, Queen's University, Kingston, ON K7L 3N6
*Weak high order Maximum Principle for control-affine systems and applications*

An optimal control problem from the differential geometric viewpoint consists of finding integral curves of vector fields depending on parameters, called controls, satisfying end-point conditions and minimizing a particular functional. High order Maximum Principle (Krener, 1977) provides necessary conditions for finding optimal solutions. These conditions include the first order necessary conditions in Pontryagin's Maximum Principle.

Here, we state high order Maximum Principle for control-affine systems in a weaker way by means of a presymplectic equation. This equation starts a presymplectic constraint algorithm in the sense of Gotay–Nester–Hinds. We establish the connections between the presymplectic constraint algorithm and the candidates to be optimal curves obtained from the necessary conditions in high order Maximum Principle. These connections are obtained by means of the high order perturbation cones that contain the vectors that approximate conveniently all the perturbations of a reference trajectory.

An optimal control problem has different kinds of solutions: the normal and the abnormal ones. As application of the above results, we characterize the abnormal extremals, i.e., curves candidates to be optimal, for control mechanical systems described by affine connections. The peculiarity of abnormality is that, in a first approach, does not depend on the cost function whose functional needs to be minimized.

**MICHAEL CAVERS**, University of Regina
*On the normalized Laplacian energy of a graph*

The concept of graph energy was defined by Ivan Gutman in 1978 and originates from theoretical chemistry. To determine the energy of a graph, we essentially add up the eigenvalues (in absolute value) of the adjacency matrix of a graph. Recently, a few analogous quantities of energy have been defined, including the *Laplacian energy* and *distance energy*. In this talk, we analyze the *normalized Laplacian energy* of a graph, called $\mathcal{L}$-energy, defined as

$$E_{\mathcal{L}}(G) = \sum_{i=1}^{n} |\lambda_i(\mathcal{L}) - 1|,$$

where $\lambda_1(\mathcal{L}), \dots, \lambda_n(\mathcal{L})$ are the eigenvalues of the normalized Laplacian matrix $\mathcal{L}$ of a graph $G$. We highlight some results on the $\mathcal{L}$-energy of graphs and relate it to a known topological index called the general Randić index. We will analyze how the $\mathcal{L}$-energy relates to the standard energy and how the structure of a graph affects $\mathcal{L}$-energy.

**MARTIN MULDOON**, York University, Toronto, ON M3J 1P3
*Monotonic sequences related to zeros of Bessel functions*

In the course of their work on Salem numbers and uniform distribution modulo $1$ (see Publ. Math. Debrecen **64**(2004), 329–341), A. Akiyama and Y. Tanigawa proved some inequalities concerning the values of the Bessel function $J_0$ at multiples of $\pi$, i.e., at the zeros of $J_{1/2}$. This raises the question of inequalities and monotonicity properties for the sequences of values of one cylinder function at the zeros of another such function. Here we derive such results by differential equations methods. Some related results are considered.

This is joint work with Lee Lorch.

**SRIYANI RATHUGAMAGE**,
*A Membrane in Dynamic Adhesive Contact*

We analyze a model for dynamic adhesive contact between a stretched viscoelastic membrane and a reactive obstacle that lies beneath it. The model consists of a hyperbolic equation for the vibrations of the membrane coupled with a nonlinear ordinary differential equation for the evolution of the bonding function.

The new feature in the model is the choice of the adhesion rate exponent which allows for complete debonding of the membrane from the support in finite time.

The existence of the unique weak solution for the model with viscosity is established using a sequence of approximate and truncated problems, a priori estimates on their solutions, and a fixed point argument. The solutions are found to be quite regular. We also use these estimates to pass to the limit of vanishing viscosity and obtain a weak solution of the inviscid problem.

**N. BRAD WILLMS**, Bishop's University, 2600 College St., Sherbrooke, QC J1M 1Z7
*A Geometric Necessary Condition for the Schiffer Problem*

The Schiffer problem is an overdetermined problem for the Laplace operator: find connected, bounded domains on which some non-constant eigenfunction under homogeneous Neumann boundary conditions, is *also* constant on the boundary. The conjecture due to M. M. Schiffer is that the only simply-connected domains possessing solutions are $n$-balls. Such domains are said to fail the Schiffer property. We derive a purely geometric condition, necessarily satisfied by the boundary curve of any two-dimensional domain failing the Schiffer property. Schiffer's problem then becomes a problem of differential geometry in the large.