
MARIA BARBERO, Queen's University, Kingston, ON K7L 3N6

Weak high order Maximum Principle for control-affine systems and applications

An optimal control problem from the differential geometric viewpoint consists of finding integral curves of vector fields depending on parameters, called controls, satisfying end-point conditions and minimizing a particular functional. High order Maximum Principle (Krener, 1977) provides necessary conditions for finding optimal solutions. These conditions include the first order necessary conditions in Pontryagin's Maximum Principle.

Here, we state high order Maximum Principle for control-affine systems in a weaker way by means of a presymplectic equation. This equation starts a presymplectic constraint algorithm in the sense of Gotay–Nester–Hinds. We establish the connections between the presymplectic constraint algorithm and the candidates to be optimal curves obtained from the necessary conditions in high order Maximum Principle. These connections are obtained by means of the high order perturbation cones that contain the vectors that approximate conveniently all the perturbations of a reference trajectory.

An optimal control problem has different kinds of solutions: the normal and the abnormal ones. As application of the above results, we characterize the abnormal extremals, i.e., curves candidates to be optimal, for control mechanical systems described by affine connections. The peculiarity of abnormality is that, in a first approach, does not depend on the cost function whose functional needs to be minimized.