Representation Theory of Algebras
Théorie des représentations des algèbres
(Org: **Vlastimil Dlab (Carleton)** (Carleton) and/et **Ragnar-Olaf Buchweitz** (Toronto))

YU A. BACHTURIN, Memorial University of Newfoundland
Finite-dimensional Graded Simple Algebras

I am going to talk about our joint results with Sudarshan Sehgal and Mikhail Zaicev on the structure of finite-dimensional graded simple algebras. These algebras have been described in term of twisted group rings and matrix algebras with fine and elementary gradings.

MARGARET BEATTIE, Mount Allison University, Sackville, NB, E4L 1E6
On the classification of low dimensional Hopf algebras

The general classification of finite dimensional Hopf algebras over a field of characteristic 0 is a difficult open problem. In the study of Hopf algebras of low dimension which are neither cosemisimple nor pointed, one approach is to study the simple coalgebras in the coradical H_0 of the Hopf algebra H and the H_0-bicomodules P_n. Note that this amounts to studying the irreducible representations of the dual Hopf algebra H^*. We outline some older and newer results for this problem.

FRAUKE M. BLEHER, University of Iowa, Department of Mathematics, 14 MLH, Iowa City, IA 52242-1419, USA
Universal deformation rings and tame blocks

Let k be an algebraically closed field of positive characteristic, and let G be a finite group. There are various classical results in the literature concerning the lifting of finitely generated kG-modules over complete discrete valuation rings, such as Green's liftability theorem. To understand and generalize these results, it is useful to reformulate them in terms of deformation rings.

Suppose B is a block of kG of tame representation type with defect group D. For certain B, we will show how to determine the universal deformation rings $R(G,V)$ of finitely generated kG-modules V belonging to B which have stable endomorphism ring isomorphic to k. We will relate $R(G,V)$ to the group ring WD where W is the ring of infinite Witt vectors over k.

CHRISTOPHER BRAV, University of Toronto
The projective McKay correspondence

Kirillov has described a geometric McKay correspondence for finite subgroups $G \subset \mathrm{PSL}_2(\mathbb{C})$: for each 'height function' on the affine Dynkin diagram associated to G, there is a derived equivalence from G-equivariant sheaves on $\mathbb{P}^1$ to the path algebra of an orientation of the diagram. These equivalences for various height functions are related by reflection functors.

I develop an analogous McKay correspondence for the cotangent bundle $T^*\mathbb{P}^1$ in which each height function gives a derived equivalence from equivariant sheaves on $T^*\mathbb{P}^1$ to the preprojective algebra of the affine Dynkin diagram. These various equivalences are related by so-called spherical twists, which generate an action of the Artin group of the diagram on the derived category of equivariant sheaves.

THOMAS BRUESTLE, Bishop's University and Université de Sherbrooke
From Christoffel words to Markoff numbers

For a pair (a,b) of relatively prime natural numbers, the Christoffel word $C(a,b)$ is defined by the path with integral vertices which is closest to the line segment from $(0,0)$ to (a,b). Viewing this line segment as an arc in the once-punctured torus, we define a J-module $M(a,b)$ for each Christoffel word. Here J is the Jacobian algebra of the once-punctured torus. We show that one obtains the Markoff number associated with $C(a,b)$ by counting submodules of $M(a,b)$.

RAGNAR-OLAF BUCHWEITZ, University of Toronto at Scarborough, UTSC, 1265 Military Trail, Toronto, ON, M1C 1A4
On the Endomorphismring of the Syzygies in the Tautological Koszul Complex

Recent work on noncommutative desingularisations of determinantal varieties led us to take a closer look at the object in the title. While its linear properties, such as local cohomology and projective resolution, are well within reach of a first course in homological algebra—and provide interesting examples—the multiplicative properties are more intriguing.

Based on results by T. Bridgeland, we know that the n-th Veronese subalgebra, if n is the number of variables, is Koszul, Calabi–Yau, of finite global dimension, and provides an algebraic model of the anti-canonical bundle of the underlying projective space, realising that bundle as a moduli space of representations of that algebra.

We will indicate how these results relate to the theory of quiver representations and helices.

Joint work with Thuy Pham.

VLASTIMIL DLAB, Carleton University, Ottawa, ON, K1S 5B6
Endomorphism algebras of generator-cogenerator modules over hereditary Artin algebras

The lecture will provide a complete description of the possibilities for global dimension of the endomorphism algebras described in the title in terms of the cardinalities of the Auslander–Reiten orbits of indecomposable modules.

A joint result with C. M. Ringel.

FRANCOIS HUARD, Bishop's

SHIPING LIU, Université de Sherbrooke
On almost projective and almost injective modules

Let A be an artin algebra, and consider the category $\operatorname{mod} A$ of finitely generated right A-modules. A module M in $\operatorname{mod} A$ is called *almost projective* if $\operatorname{Ext}^1_A(M,X) \neq 0$ for at most finitely many non-isomorphic indecomposable modules in $\operatorname{mod} A$; and *almost injective* if $\operatorname{Ext}^1_A(X,M) \neq 0$ for at most finitely many non-isomorphic indecomposable modules in $\operatorname{mod} A$. We shall show that the almost projective or injective modules are distributed in finitely many DTr-orbits in the Auslander–Reiten quiver of A. In particular, A is of finite representation type if and only if every module in $\operatorname{mod} A$ is almost projective or injective. As a consequence, if A is a finite-dimensional algebra over an algebraically closed field of infinite representation type, then there exists infinitely many non-isomorphic indecomposable modules in $\operatorname{mod} A$ which are neither almost projective nor almost injective.

FRANK MARKO, The Pennsylvania State University Hazleton
Structure of costandard and simple modules for Schur superalgebras in positive characteristics

We describe recent results for Schur superalgebras in positive characteristics. We discuss similarities and differences of the structure of simple and costandard modules for Schur superalgebras and for classical Schur algebras. In particular, we consider the structure of these modules for Schur superalgebras $S(2|1)$ and $S(2|2)$.

FRANK ZORZITTO, University of Waterloo, Waterloo, Ontario
Endomorphism algebras of rank-two Kronecker modules

If A is the Kronecker algebra $\left[\begin{smallmatrix} K & K^2 \\ 0 & K \end{smallmatrix}\right]$ over an algebraically closed field K, we examine the possible endomorphism algebras of those modules M that are extensions of *finite*-dimensional, rank-one A-modules N by *infinite*-dimensional, rank-one A-modules P. The primary tool is a regulator-polynomial $f(Y)$ in the indeterminate Y having coefficients in the rational function field $K(X)$. If $\operatorname{End} M$ contains endomorphisms other than scalars from K, then $f(Y)$ is linear or quadratic in Y. Supposing $\operatorname{End} M$ properly contains K, the regulator $f(Y)$ is quadratic if and only if $\operatorname{End} M$ is commutative. This latter case has led to some intriguing results. For instance, if the quadratic $f(Y)$ has a repeated root in $K(X)$, then $\operatorname{End} M$ is isomorphic to a trivial extension $K \ltimes S$ for some K-linear space S. If $f(Y)$ has no roots in $K(X)$, then $\operatorname{End} M$ is Noetherian with zero radical. If, in addition, $\operatorname{End} P$ is an affine K-algebra, then $\operatorname{End} M$ is also affine. Hence such $\operatorname{End} M$ are the coordinate rings of some curves. If $f(Y)$ has distinct roots in $K(X)$, then $\operatorname{End} M$ is isomorphic to a fibre-product of two subalgebras of $K(X)$. In case P is the unique, torsion-free, divisible, indecomposable A-module and $f(Y)$ is quadratic, $\operatorname{End} M$ properly contains K if and only if M is decomposable. Such results invite generalization to representations of finite-dimensional algebras other than the Kronecker algebra.