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A variational principle associated to a certain class of boundary value problems

A variational principle is introduced to provide a new formulation and resolution for several boundary value problems. Indeed, we consider systems of the form

$$\begin{cases} \Lambda u = \nabla\Phi(u), \\ \beta_2 u = \nabla\Psi(\beta_1 u) \end{cases}$$

where Φ and Ψ are two convex functions and Λ is a possibly unbounded self-adjoint operator modulo the boundary operator $\mathcal{B} = (\beta_1, \beta_2)$. We shall show that solutions of the above system coincide with critical points of the functional

$$I(u) = \Phi^*(\Lambda u) - \Phi(u) + \Psi^*(\beta_2 u) - \Psi(\beta_1 u)$$

where Φ^* and Ψ^* are Fenchel–Legendre dual of Φ and Ψ respectively. Note that the standard Euler–Lagrange functional corresponding to the system above is of the form,

$$F(u) = \frac{1}{2}\langle \Lambda u, u\rangle - \Phi(u) - \Psi(\beta_1 u).$$

An immediate advantage of using the functional I instead of F is to obtain more regular solutions and also the flexibility to handle boundary value problems with nonlinear boundary conditions. Applications to Hamiltonian systems and semi-linear elliptic equations with various linear and nonlinear boundary conditions are also provided.