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Improving the stability of numerical simulations of quantum dynamical systems using stochastic differential equation techniques

Numerical simulations of path integrals by stochastic differential equations (SDEs) sometimes show stability problems. An example of an ill-posed problem is the case of one-mode Bose–Einstein condensation in a coherent state representation [1]. In this case, the numerical solution of the resulting SDE becomes unstable after a relatively short time for most numerical methods and can blow up in finite time if uncontrolled.

To improve the results, new numerical methods are being developed, but it appears that the choice of the SDE integrator alone is insufficient to guarantee stability. One reason for this is that the drift depends on a conformal martingale which can have arbitrary phase and amplitude. Furthermore, the related Fokker–Planck equation of the problem turns out to be of mixed type with hyperbolic regions. It appears that regularization techniques combined with implicit SDE solvers and extrapolation methods can yield significant stability improvements.

References

[1] P. Deuar and P. D. Drummond, *Stochastic gauges in quantum dynamics for many-body simulations.* Comput. Phys. Commun. **142**(2001), 442–445.