Mathematics and Classical Mechanics
Mathématique et mécanique classique
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STEPHEN ANCO, Brock University, St. Catharines, ON, L2S 3A1
Klein geometry and bi-Hamiltonian structure of soliton equations

From the viewpoint of classical mechanics, a distinguishing feature of soliton equations is that they possess two compatible Hamiltonian formulations. In the case of the sine-Gordon (SG) equation, modified Korteweg–de Vries (mKdV) equation, and nonlinear Schrodinger (NLS) equation, it is known that their bi-Hamiltonian structure has a remarkable geometric origin connected with the classical frame structure equations for curve motions in S^2, R^2, and R^3, respectively. In this talk I will describe a broad generalization of these results to the setting of curve motions in Klein geometry, which gives a geometrical derivation of group-invariant (multicomponent) generalizations of mKdV, NLS, and SG soliton equations along with their bi-Hamiltonian structure, symmetries, and conservation laws.

LARRY BATES, University of Calgary, Calgary, AB
No monodromy in the champagne bottle

It is a curiosity that the three dimensional champagne bottle problem contains no monodromy, despite the fact that it is composed of two-dimensional problems that all contain monodromy. This talk will explain 'where the monodromy went'.

LEO BUTLER, University of Edinburgh, School of Mathematics, James Clerk Maxwell Bldg., King's Buildings, Edinburgh, UK, EH9 3JZ
The Maslov cocycle, smooth structures and real-analytic complete integrability

In this talk, I will discuss two main results. First, I show that if the cotangent bundle of a smooth manifold homeomorphic to the standard n-torus admits a real-analytically completely integrable convex hamiltonian, then the manifold is diffeomorphic to the standard n-torus. Second, I prove that for some topological 7-manifolds, the cotangent bundle of each smooth structure admits a real-analytically completely integrable riemannian metric hamiltonian.

This proves that the existence of a real-analytically completely integrable convex hamiltonian is a non-trivial smooth invariant of a manifold.

ALEXEI CHEVIAKOV, University of Saskatchewan
Nonlocally related PDE systems and exact solutions for one-dimensional nonlinear elastodynamics

We start from a derivation of complete dynamical PDE systems of one-dimensional nonlinear elasticity satisfying the principle of material frame indifference, in Eulerian and Lagrangian formulations. We then consider these systems within the framework of nonlocally related PDE systems, and derive *a direct relation* between the Euler and Lagrange systems within that framework. Moreover, other equivalent PDE systems *nonlocally related* to both of these familiar systems are obtained.

Point symmetries of three of these nonlocally related PDE systems of nonlinear elasticity are classified with respect to constitutive and loading functions. Consequently, new symmetries are computed that are: nonlocal for the Euler system and local for the Lagrange system; local for the Euler system and nonlocal for the Lagrange system; nonlocal for both the Euler and Lagrange systems.

For realistic constitutive functions and boundary conditions, we use the obtained nonlocal symmetries to construct new exact dynamical solutions, and prove that they *do not* arise from invariance under local symmetries.

This is a joint work with G. Bluman (UBC) and J.-F. Ganghoffer (INPL, Nancy, France).

FLORIN DIACU, University of Victoria
The n-body problem in spaces of constant curvature

We generalize the Newtonian n-body problem to spaces of curvature $k =$ constant, and study the motion in the 2-dimensional case. For $k > 0$, the equations of motion encounter non-collision singularities, which occur when two bodies are antipodal. This phenomenon leads, on one hand, to hybrid solution singularities for as few as 3 bodies, whose corresponding orbits end up in a collision-antipodal configuration in finite time; on the other hand, it produces non-singularity collisions, characterized by finite velocities and forces at the collision instant. We also point out the existence of several classes of relative equilibria, including the hyperbolic rotations for $k < 0$. In the end, we prove Saari's conjecture when the bodies are on a geodesic that rotates elliptically or hyperbolically. We also emphasize that fixed points are specific to the case $k > 0$, hyperbolic relative equilibria to $k < 0$, and Lagrangian orbits of arbitrary masses to $k = 0$—results that provide new criteria towards understanding the large-scale geometry of the physical space.

ANTONIO HERNANDEZ-GARDUNO, Universidad Nacional Autónoma de México
Stability of Lagrangian relative equilibria for the roto-translational three-body problem

A relative equilibrium in the N-body problem is a solution for which the whole system rotates (rigidly) around a fixed axis with constant angular velocity. Usually one thinks of the bodies as point masses interacting under a Newtonian potential. Nevertheless, perturbations to this potential arise when one considers the three-dimensional aspect of the bodies. In this talk we discuss the application of the reduced energy-momentum method to the study of the stability of the roto-translational three-body problem, with perturbations to the Newtonian potential due to oblateness.

This is a joint work with Cristina Stoica.

JACQUES HURTUBISE, McGill University, 80 Sherbrooke St., Montreal, QC, H3A 2K6
The geometry of isomonodromic deformations

This talk will examine the geometry behind the Hamiltonian structure of isomonodromy deformations of connections on vector bundles over Riemann surfaces. The main point is that one should think of an open set of the moduli of pairs (V, ∇) of vector bundles and connections as being obtained by "twists" supported over points of a fixed vector bundle V_0 with a fixed connection ∇_0; this gives two deformations, one, isomonodromic, of (V, ∇), and another induced from the isomonodromic deformation of (V_0, ∇_0). The difference between the two will be Hamiltonian.

JEROEN LAMB, Imperial College London
Branching patterns of wave trains in the FPU lattice

We study the existence and branching patterns of wave trains in the one-dimensional infinite Fermi–Pasta–Ulam (FPU) lattice. A wave train Ansatz in this Hamiltonian lattice leads to an advance-delay differential equation on a space of periodic functions, which carries a natural Hamiltonian structure. The existence of wave trains is then studied by means of a Lyapunov–Schmidt reduction, leading to a finite-dimensional bifurcation equation with an inherited Hamiltonian structure. While exploring some of the additional symmetries of the FPU lattice, we use invariant theory to find the bifurcation equations describing the branching patterns of wave trains near $p : q$ resonant waves. We show that at such branching points, a generic nonlinearity selects exactly two two-parameter families of mixed-mode wave trains.

This is joint work with Bob Rink (Free University Amsterdam) and Shangjiang Guo (University of Hunan).

BILL LANGFORD, University of Guelph, Guelph, ON, N1G 2W1
Poleward Expansion of Hadley Cells

Recent reanalyses of meteorological data by climate scientists have indicated that the Hadley cells of the atmospheric circulation are expanding toward the poles as well as slowing in their circulation velocity. A majority of GCM simulations forecast that these trends will continue at least to the end of the 21st century. If true, the poleward expansion of Hadley cells would lead to desertification of economically important regions. Similar reanalyses of meteorological data for recent decades show a poleward movement of the jet streams that affect midlatitude weather. Although the precise mechanism of these changes in Hadley cells and jet streams is not fully understood, it is believed to be linked with global warming. In this paper, we apply pseudo-arclength continuation to a model of a fluid in a differentially heated rotating spherical shell that uses the Navier–Stokes equations in the Boussinesq approximation. We demonstrate that a decrease in the pole-to-equator temperature gradient leads to an expansion and slowing of the Hadley circulation and a poleward movement of jet streams, thus indicating a possible mechanism for the observed changes.

Joint work with G. Lewis, UOIT.

TSUNG-LIN LEE, Michigan State University
The polyhedral homotopy continuation method and its applications to celestial mechanics

While the classical linear homotopy continuation method was developed for solving polynomial systems in the 1980s, the polyhedral homotopies are established in 1995, which yields a drastic improvement over the classical linear homotopies. HOM4PS-2.0 is a software package which implements the polyhedral homotopy continuation method for solving polynomial systems. With several sophisticatedly designed algorithms in mixed cell computation and curve tracing, it surpasses the existing packages in finding isolated zeros of polynomial systems, such as PHCpack, PHoM, and Bertini, in speed by big margins. With the marvelous efficiency of HOM4PS-2.0, it is now possible to solve some very large systems. Its applications to celestial mechanics will be presented in this talk.

RAY McLENAGHAN, University of Waterloo, Department of Applied Mathematics, Waterloo, Ontario N2L 4B7
Invariant classification of the orthogonally separable webs in Minkowski 3-space

An invariant classification of the orthogonally separable webs for the Hamilton–Jacobi equation for the geodesics and the wave equation in terms of invariants of valence two Killing tensors under the action of the isometry group is described. The theory is applied to show that the Morosi–Tondo integrable system derived as a stationary reduction of the seventh order Korteweg–de Vries flow is an orthogonally separable Hamiltonian system.

Joint work with Joshua Horwood and Roman Smirnov.

GEORGE PATRICK, University of Saskatchewan
Local error analysis of variational integrators

Due to a singularity or degeneracy at zero time-step, existence and uniqueness, and accuracy, of variational integrators, cannot be established by straightforward use of the implicit function theorem. We show existence and uniqueness for variational integrators by blowing up the associated discrete variational principle. The blow-up leads to an accuracy one less than is observed in simulations, a deficit that is recovered by a past-future symmetry at zero time-step.

GARETH ROBERTS, College of the Holy Cross, 1 College Street, Worcester, MA 01610, USA
Using BKK Theory in Restricted N-Body Problems

The purpose of this talk is to demonstrate the effectiveness of Bernstein–Khovanskii–Kushnirenko (BKK) theory for solving important finiteness problems in celestial mechanics. In particular, we show that it is not possible for a solution of the planar, circular, restricted three-body problem (PCR3BP) to travel along a level curve of the amended potential without being fixed at one of the five libration points (Saari's conjecture for the PCR3BP). Equivalently, the only orbits traveling with constant speed are equilibria. We also use BKK theory to show that, for any choice of masses, the number of equilibria in the PCR4BP is finite and bounded above by 196.

ROMAN SMIRNOV, Dalhousie University
Joint invariants and resultants of Killing tensors in classical mechanics

We introduce the notion of a resultant of Killing tensors (KTs) in the general framework of the invariant theory of Killing tensors. Just like in the classical invariant theory of homogeneous polynomials it is based on the corresponding notion of a joint invariant. Using the properties of resultants of KTs we derive a new geometric (or, equivalently, algebraic) characterization of the Kepler potential in classical mechanics.

This is joint work with Caroline Adlam and Ray McLenaghan.

EDWARD THOMMES, Department of Physics, University of Guelph
Modelling the birth of planetary systems with a hybrid symplectic integrator

Symplectic N-body maps have lead to a major leap forward in the numerical study of the Solar System and extrasolar planetary systems. Such methods conserve energy even with a relatively small number of timesteps per dynamical time, thus enabling simulations which span the entire lifetime of a planetary system. I will review past work and describe a new hybrid N-body integrator which is providing us with an unprecedented level of insight into the birth process of planetary systems, including our own.

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