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*De Moivre's Proof of De Moivre's Identity*

In his work *Miscellanea Analytica* of 1730, Abraham De Moivre provided a detailed derivation of the following result. If  $x$  is the cosine of an arc and  $l$  is the cosine of an arc  $n$  times this arc then  $x$  and  $l$  are related by the identity

$$x = \frac{1}{2} \sqrt[n]{l + \sqrt{l^2 - 1}} + \frac{\frac{1}{2}}{\sqrt[n]{l + \sqrt{l^2 - 1}}}. \quad (1)$$

It is evident that De Moivre possessed a knowledge of what is known today as DeMoivre's identity

$$(\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin \theta. \quad (2)$$

De Moivre had presented (1) without proof in 1707 in an article in the *Philosophical Transactions*. We provide an account of De Moivre's 1730 derivation, focusing on a critical evaluation of the proof. De Moivre's intent was to ground the proof in the nature of the calculus as it was understood at this time, which was as a set of algorithms and operations on variables that enabled one to relate and to calculate various geometric quantities associated with the curve. He was uneasy with procedures involving the integration of imaginary expressions. Viewed formally in terms of operations and transformations, such procedures were feasible and led to useful results. Nevertheless, they lacked any interpretation within the established geometric formulation of the calculus. De Moivre desired a deduction that was grounded in known foundational conceptions, and the end result was the unusual derivation presented in the *Miscellanea Analytica*.