Dynamics of Large Groups and Semigroups
Propriétés dynamiques des groupes et des demi-groupes de dimension infinie
(Org: **Stefano Ferri** (Uniandes, Bogotá), **Alica Miller** (Louisville) and/et **Vladimir Pestov** (Ottawa))

CHRISTOPHER ATKIN, Victoria University of Wellington, New Zealand
Boundedness in some groups of homeomorphisms

The principal component of the group of homeomorphisms of a compact manifold M is sometimes bounded and sometimes unbounded in the compact-open topology; some suggestive examples and counterexamples will be presented.

ALEXANDER BERENSTEIN, Universidad de los Andes–Université Claude Bernard Lyon 1
Actions of groups on probability measure algebras: a model theory perspective

For a countable amenable group G, we consider probability measure algebras equipped with a measure preserving action of G from the perspective of continuous logic. We characterize the actions corresponding to the class of existentially closed separable structures as the ones arising from free actions of G on Lebesgue spaces. We also show that for a non-amenable group G there are free actions by measure preserving automorphisms on a probability space that do not induce existentially closed structures.

This is joint work with Ward Henson.

MOHAMMED DARRAS, University of Ottawa
Kahzadan's property (T) for some infinite dimensional topological groups

According to a theorem of Bekka, the unitary group of an infinite dimensional separable Hilbert space equipped with strong operator topology has Kahzdan's property (T). We will analyze Bekka's proof and we will extract a proof of a more general result. One of the corollaries is a new theorem that the infinite symmetric group equipped with natural Polish topology has Kahzdan's property (T); the proof of this theorem uses a result of Liebarman. After that we will discuss some interesting open questions.

This is part of a future PhD thesis written under supervision of Vladimir Pestov and David Handelman.

ILIAS FARAH, York University
Ultrapowers of unitary groups of UHF algebras

It is well known that the Continuum Hypothesis implies all ultrapowers of a fixed Polish group G are isomorphic. (This is provided that all ultrafilters we consider are nonprincipal ultrafilters on the set of natural numbers.) I will show that the converse holds if G is the unitary group of a UHF C^*-algebra: If the Continuum Hypothesis fails, then there are nonisomorphic ultrapowers of G. The analogous statement for the relative commutant of G in the ultrapower is also true. Its variant for C^*-algebras answers a question of Kirchberg. I will also give some remarks on the extreme amenability of the unitary groups of AF and UHF C^*-algebras.

STEFANO FERRI, Universidad de los Andes
Large free semigroups in the WAP-compactification

A digital representation of a semigroup $(S,\cdot)$ is a family $\langle F_t\rangle_{t\in I}$, where I is a linearly ordered set, each F_t is a finite non-empty subset of S and every element of S is uniquely representable in the form $\prod_{t\in H} xt$ where H is a finite subset of I, each $xt \in F_t$ and products are taken in increasing order of indices. (If S has an identity 1, then $\prod_{t\in\emptyset} xt = 1$.)

We use digital representation to show that if G is an Abelian group with cardinality κ, then the Weakly Almost Periodic compactification of G contains a copy of a free Abelian semigroup of cardinality 2^{2^κ}.

This is joint work with Neil Hindman and Dona Strauss.

MATTHEW FOREMAN, UC Irvine
Models for the space of measure-preserving systems

The group of invertible measure-preserving transformations of the unit interval contains isomorphs of every invertible measure-preserving transformation on a separable probability space. Dually, the collection of shift invariant measures on $X^{\mathbb{Z}}$ is another universal model. Are they equivalent? Are there other non-equivalent models? What does equivalence mean? Rudolph has conjectured that all models are equivalent, and work of Glasner and King supports this conjecture.

In joint work with B. Weiss, I will discuss some new models for measure-preserving transformations and ergodic measure-preserving transformations and present some general machinery for showing equivalence.

JORGE GALINDO, Universidad Jaume I, Castellón, Spain
On the Eberlein compactification of a topological group

The set $B(G)$ of all matrix coefficients associated to continuous unitary representations of a given topological group G is a $*$-closed, subalgebra of the C^*-algebra $\ell_\infty(G)$ of all bounded functions on G. We will refer to the spectrum of the closure (in $\ell_\infty(G)$) of this subalgebra as the *Eberlein compactification* eG of G.

Multiplication on G can be extended in the standard way to the Eberlein compactification and eG is thus made into a semitopological semigroup. eG is therefore a semigroup compactification of G placed between the almost periodic and weakly almost periodic compactifications. In this talk we will overview some of the features of the Eberlein compactification eG that concern its size, complexity, algebraic structure or the way G fits in it.

WADII HAJJI, Ottawa University
Compact inverse semigroups

W. D. Munn proved that a finite dimensional representation of an inverse semigroup is equivalent to a partial unitary representation if and only if it is bounded. The first goal of this talk will be to give new analytic proof that every finite dimensional representation of a compact inverse semigroup is equivalent to a partial unitary representation.

The second goal is to parameterize all finite dimensional irreducible representations of a compact inverse semigroup in terms of maximal subgroups and order theoretic proprieties of the idempotent set. As a consequence, we obtain a new and simple proof of the following theorem of Shneperman: a compact inverse semigroup has enough finite dimensional irreducible representations to separate points if and only if its idempotent set is totally disconnected.

WOJCIECH JAWORSKI, Carleton University
Dissipation of Convolution Powers in a Metric Group

In contrast to what is known about probability measures on locally compact groups, a metric group G can support a probability measure μ which is not carried on a compact subgroup but for which there exists a compact subset $C \subseteq G$ such that the sequence $\mu^n(C)$ fails to converge to zero as n tends to ∞. A noncompact metric group can also support a probability measure μ such that $\operatorname{supp}\mu = G$ and the concentration functions of μ do not converge to zero. We will discuss a number of conditions which guarantee that the concentration functions in a metric group G converge to zero. We will also present a sufficient and necessary condition in order that a probability measure μ on G satisfy $\lim_{n\to\infty} \mu^n(C) = 0$ for every compact subset $C \subset G$.

CLAUDE LAFLAMME, University of Calgary, Alberta, Canada

Ramsey Theory on Vector Spaces

We discuss infinite versions of vector space partitions due to Graham–Leeb–Rothschild. Hindman observed long ago that if V is a vector space over $GF(2)$ with countable dimension then V is indivisible, meaning that for every partition of V into two parts, one of the parts contains an affine copy of V. Together with L. Nguyen Van Thé, M. Pouzet and N. Sauer, we have shown that if V is a vector space over a field different from $GF(2)$ then V is not indivisible: in fact it does not have canonical partitions. If the field is infinite, we can show even more, the vector space can be divided into two parts such that none of the parts contains an affine line. The is provides some insight as to the connection between age and weak indivisibility.

GABOR LUKACS, University of Manitoba, Winnipeg, MB

Locally precompact groups

A subset B of a (Hausdorff) topological group G is said to be *precompact* if for every neighborhood U of the identity in G, there is a finite subset $F \subseteq G$ such that $B \subseteq (FU) \cap (UF)$. An interesting subclass of the class of precompact groups was identified and studied by Comfort and Ross, who showed inter alia that a topological group G is pseudocompact if and only if it is precompact and G_δ-dense in its completion $\widetilde{G}$ (cf. [2]). Since then, precompact groups have been a focus of interest (cf. [4], [1]).

A group G is *locally precompact* if it contains a precompact neighborhood of the identity. The completion of a locally precompact group is locally compact (cf. [6]), and thus such groups are precisely the subgroups of locally compact groups. Comfort and Trigos-Arrieta extended the Comfort–Ross criterion, and proved that a locally precompact group G is locally pseudocompact if and only if it is G_δ-dense in $\widetilde{G}$ (cf. [3]). Locally pseudocompact groups were also studied by Sanchis (cf. [5]).

In this talk, we discuss the relationship between cardinal invariants of locally precompact groups and completeness properties such as realcompactness.

References

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JAN PACHL,

Ambitable groups and semigroups

For ambitable topological groups we have a tractable characterization of topological centres in certain convolution algebras [arXiv:0803.3405]. The same holds more generally for ambitable semigroups, defined as a subclass of so-called semiuniform semigroups. Hence the question: Which semiuniform semigroups are ambitable? For groups, the question is close to being completely answered: With a possible exception of certain "singular" groups, a topological group is ambitable if and only if it is not precompact. In particular, every locally $\aleph_n$-bounded group, $n = 1, 2, \ldots$, is either precompact or ambitable. In contrast, little is known about the question for semigroups beyond groups.

VLADIMIR PESTOV, University of Ottawa, Ontario, Canada
Amenability test spaces for Polish groups

A compact space X is an *amenability test space* for a class $\mathcal{C}$ of topological groups if a group $G \in \mathcal{C}$ is amenable if and only if every continuous action of G on X admits an invariant Borel probability measure. De la Harpe and Giordano (C. R. Acad. Sci. Paris **324**(1997), 1255–1258), answering a question of Grigorchuk, had proved that the Cantor space C is an amenability test space for discrete countable groups. Bogatyi and Fedorchuk (Topol. Methods Nonlinear Anal. **29**(2007), 383–401) had obtained the same conclusion for the Hilbert cube Q. It remains unknown whether the Menger compacta serve as amenability test spaces for discrete countable groups. With my M.Sc. student Yousef Al-Gadid, we had shown that the Cantor space is a test space for *topological amenability,* also known as *amenability at infinity,* of countable discrete groups.

Jointly with Brice Rodrigue Mbombo Dempowo (Université de Yaoundé, Cameroun), we had observed that the space C remains an amenability test space for every Polish group with small open subgroups, while the Hilbert cube Q is a test space for every Polish group. At the same time, we do not know whether there exists a compact metrizable test space X for detecting *extreme amenability* of Polish groups, in other words, having the property that a Polish group G has a fixed point in every compact space upon which it acts continuously whenever it has fixed point whenever it acts continuously on X. A separable compact X with this property exists for cardinality considerations, but, for instance, Q is not such in view of Shauder's theorem, implying that every action of $\mathbb{Z}$ on Q has a fixed point.

NORBERT SAUER, University of Calgary, 2400 University Dr. NW, Calgary
Partitions of permutation groups

Let $\mathcal{H}$ be an ultrahomogeneous structure on a set R. The properties of $\mathcal{H}$ to be *age indivisible, weakly indivisible, has a canonical partition, is indivisible, the age is a Ramsey class, or the distinguishing number,* etc., are actually properties of the automorphism group of $\mathcal{H}$. On the other hand if G is a group of bijections on R closed in the discrete topology then there is an ultrahomogeneous structure $\mathcal{H}$ on R with automorphism group G. The partition properties alluded to above can be directly defined in permutation theoretic terms. If G then acts as a topological group on a compact Hausdorff space or $\mathcal{H}$ is the group of isometries of a homogeneous metric space on R then some of the partition properties of $\mathcal{H}$ or of structures related to $\mathcal{H}$ imply topological properties of the group.

The relation between the various partition properties is in one direction quite easy to establish but poses an interesting problem in several of the other directions.

LIONEL NGUYEN VAN THÉ, University of Calgary, 2500 University Drive NW, Calgary, Alberta, T2N 1N4
The universal minimal flows of $S(2)$ and $S(3)$

For a topological group G, a compact minimal G-flow is a compact Hausdorff space X together with a continuous action of G on X for which the orbit of every point is dense in X. It is a general result in topological dynamics that every Hausdorff topological group G has a compact minimal G-flow $M(G)$ which is, moreover, universal, in the sense that it can be mapped homomorphically onto any other compact minimal G-flow. The purpose of this talk is to show how an extension of a theorem by Kechris, Pestov and Todorcevic allows to compute the universal minimal flows of two particular groups: the automorphism groups (equipped with the pointwise convergence topology) of the dense local order $S(2)$ and of the circular directed graph $S(3)$.