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Amenability test spaces for Polish groups

A compact space X is an *amenability test space* for a class $\mathcal{C}$ of topological groups if a group $G \in \mathcal{C}$ is amenable if and only if every continuous action of G on X admits an invariant Borel probability measure. De la Harpe and Giordano (C. R. Acad. Sci. Paris **324**(1997), 1255–1258), answering a question of Grigorchuk, had proved that the Cantor space C is an amenability test space for discrete countable groups. Bogatyi and Fedorchuk (Topol. Methods Nonlinear Anal. **29**(2007), 383–401) had obtained the same conclusion for the Hilbert cube Q. It remains unknown whether the Menger compacta serve as amenability test spaces for discrete countable groups. With my M.Sc. student Yousef Al-Gadid, we had shown that the Cantor space is a test space for *topological amenability,* also known as *amenability at infinity,* of countable discrete groups.

Jointly with Brice Rodrigue Mbombo Dempowo (Université de Yaoundé, Cameroun), we had observed that the space C remains an amenability test space for every Polish group with small open subgroups, while the Hilbert cube Q is a test space for every Polish group. At the same time, we do not know whether there exists a compact metrizable test space X for detecting *extreme amenability* of Polish groups, in other words, having the property that a Polish group G has a fixed point in every compact space upon which it acts continuously whenever it has fixed point whenever it acts continuously on X. A separable compact X with this property exists for cardinality considerations, but, for instance, Q is not such in view of Shauder's theorem, implying that every action of $\mathbb{Z}$ on Q has a fixed point.