Mathematical Biology
Biologie mathématique
(Org: **Frithjof Lutscher** and/et **Robert Smith?** (Ottawa))

GERDA DE VRIES, Alberta

DAVID FISMAN, Research Institute of the Hospital for Sick Children
Gonorrhea Ain't Gone: Dissemination of Resistance via Core Groups

Gonorrhea (Gr) is a common sexually transmitted infection associated with pelvic inflammatory disease, increased susceptibility to HIV infection, and vertical transmission to neonates. The brief duration of untreated Gr, combined with low average rates of sex partner acquisition in the population, make Gr dependent on relatively small numbers of individuals with high rates of sex partner acquisition (so-called "core groups") in order to avoid extinction. Dependence on core groups represents an important vulnerability for Gr, and targeted antimicrobial treatment of these groups has proven an extremely effective means of reducing Gr incidence to very low levels throughout the population. However, rapid emergence of resistance to several antibiotic classes in recent years has been associated with a contemporaneous resurgence in Gr rates. I will use a simple, behavior-structured transmission model to show that treatment of core groups represents a highly effective strategy in the absence of resistance, but in the presence of antibiotic resistance treatment of core groups enhances dissemination of resistant strains, and undermines control efforts. In models that include two different treatment classes, a more durable reduction in Gr incidence is seen with random allocation of treatment than with sequential use of drug classes, but the preferred strategy involves use of rapid tests and targeted therapy based on microbial drug susceptibility. This model provides insights into the rapid of emergence of resistant Gr, and highlights the importance of developing point-of-care tests for antimicrobial resistance in order to maintain the advances in Gr control achieved over the past 40 years.

ELSA HANSEN, Queen's University, Kingston, Ontario
Optimal Control of Epidemics Using Limited Resources

We discuss different time-optimal control strategies for the basic *SIR* model with mass action contact rate. Among other things, the solution to an optimal control problem will depend on the cost function that the control is designed to minimize. In the literature, optimal vaccination-only policies and isolation-only policies that minimize cost functions that penalize for using control resources, have been given for the basic *SIR* model. We discuss the slightly different problem of finding optimal control strategies under the constraint of limited resources. Practically, this can be viewed as finding the best strategy, given that there are a limited amount of funds with which to implement vaccination and/or isolation. In addition to addressing this question for the vaccination-only and isolation-only models, we also present a solution for two different versions of a combined vaccination-isolation model. First we find the optimal combined policy under the assumption that the total vaccination and isolation resources have been separately allocated. Secondly, we give the solution for the case when only the total amount of resources have been allocated and the policy-maker is free to choose how to divide these resources between vaccination and isolation. For example, when planning for an epidemic, funds can be used to stockpile vaccine or prepare isolation facilities. A major advantage to using the basic *SIR* model to address these questions is that the basic forms of the solutions can be found without using numerical simulations and can often be understood using simple graphical explanations.

THOMAS HILLEN, University of Alberta, Dept. Math. Stat. Sciences
Merging and Emerging Patterns in Chemotaxis Models

The study of pattern formation for chemotaxis PDEs (partial differential equations) started with the identification of blow-up solutions. If, however, the model is adapted to allow for global existence of solutions, then another interesting pattern formation process arises. Local maxima form and they show an interaction of merging (two local maxima coagulate) or emerging (a new maximum is formed). This dynamics can lead to steady states, periodic solutions or to (what we think is) chaotic behavior. I will show that this pattern interaction is very typical for a wide variety of chemotaxis models and I will discuss possible ideas on how to understand this complicated pattern interaction.

Joint work with K. Painter and Z. Wang.

JING LI, The University of Western Ontario

An epidemic model with non-local infections on a patchy environment

With the assumptions that the infectious disease has a fixed latent period and the individuals in the latent period may disperse, we formulate an SIR model for a population living in an n-patch environment (cities, towns, or countries, etc.). The model is given by a system of delay differential equations with a fixed delay accounting for the latency and non-local terms caused by the mobility of the individuals during the latent period. An expression for the basic reproduction number $\mathcal{R}_0$ is derived, and the disease-free equilibrium is shown to be globally asymptotically stable when $\mathcal{R}_0 < 1$. At least one endemic equilibrium exists and the disease will be uniformly persistent for $\mathcal{R}_0 > 1$. For a simpler case with 2 patches only, two examples are given to illustrate that the population dispersal rate plays an important role for spread of the disease with latency, from which we can see the joint effect of the disease latency and population mobility on the disease dynamics. In addition to the existences of the disease-free equilibrium and interior endemic equilibrium, existence of boundary equilibria and their stability are also discussed in both examples.

JUSTINE GUNOG SEO, University of Ottawa, Department of Mathematics and Statistics, 585 King Edward Avenue, Ottawa, ON, K1N 6N5

A comparison of two predator-prey models with a Holling type I functional response

I analyze and compare two models, a laissez-faire and a Leslie-type predator-prey model, with Holling type I functional responses. I show, numerically, that the two models can possess two limit cycles surrounding a stable equilibrium and that these cycles arise in global cyclic-fold bifurcations. The Leslie-type model may also exhibit super-critical and discontinuous Hopf bifurcations. I then present and analyze a new functional response, built around the arctan, that smoothes the sharp corner in a type I functional response. For this new functional response, both models undergo Hopf, cyclic-fold, and Bautin bifurcations.

ROBERT SMITH?, The University of Ottawa

Determining effective spraying periods to control malaria via indoor residual spraying in sub-Saharan Africa

Indoor residual spraying—spraying insecticide inside houses to kill mosquitos—is an important method for controlling malaria vectors in sub-Saharan Africa. While it has been responsible for suppressing at least one of the malaria-carrying mosquito species, in recent years it has received relatively little attention. We propose a mathematical model for both regular and non-fixed spraying, using impulsive differential equations to account for the reduction in the mosquito population. First, we determine the stability properties of the nonimpulsive system. Next, we derive minimal effective spraying intervals and the degree of spraying effectiveness required to control mosquitos when spraying occurs at regular intervals. If spraying is not fixed, then we determine the "next best" spraying times and show that this solution is always sub-optimal. We also consider the effects of an increased mosquito birth rate on the prevalence of mosquitos. We show that, if the mosquito birth rate increased by 25%, then the minimal effective spraying period would be reduced by half, whereas if the mosquito birth rate were doubled, then the minimal effective spraying period would be reduced by three quarters. The results are illustrated with numerical solutions. It follows that, although regular spraying is superior to non-fixed spraying, either will result in a significant reduction in the overall number of mosquitos, as well as the number of malaria cases in humans. We thus recommend that the use of indoor spraying be re-examined for widespread application in malaria-endemic areas.

REBECCA TYSON, University of British Columbia Okanagan
Modelling the swimming behaviour of the nematode

How does a given aquatic organism's wiggling result in propulsion? This has been well investigated in fish and in microorganisms such as bacteria where the viscous or inertial terms of the fluid equations can be ignored. Less has been done at intermediate Reynolds' Number, and furthermore, the actual interaction between the organism's musculature and the surrounding fluid is not well understood. In this talk we focus on the swimming behaviour of the nematode, a roundworm.

The immersed boundary method lends itself very well to the study of organism locomotion in fluid. Movement of passive nematode-like structures has been successfully modeled in complex flows. Active swimming of small organisms has also been successfully modeled when the restlength of each muscle segment is prescribed, and an energy minimum for organism configuration obtained. We are interested in modelling the development of swimming motion from rest, when motion is generated by the contraction of innervated muscle segments.

We have developed a three-dimensional model for the body structure of the nematode, which explicitly models the organism's musculature. The immersed boundary method is then used to communicate between the nematode body and the surrounding fluid. This model allows us to study how the nematode musculature and surrounding fluid interact to create propulsion of the nematode. It also gives us the ability to pursue fundamental questions about how organism structure affects the swimming motion obtained and the fluid/muscle forces generated.