EDWARD ODELL, Department of Mathematics, The University of Texas at Austin, 1 University Station C1200, Austin, TX 78712-0257

Systems formed by translations of one element in $L_p(\mathbb{R})$

Let $f \in L_p(\mathbb{R})$ and for $\lambda \in \mathbb{R}$ let $f_{(\lambda)}$ be the translation of f by λ,

$$f_{(\lambda)}(t) = f(t - \lambda).$$

We study the Banach space

$$X(f, \Lambda) = [\{f_{(\lambda)} : \lambda \in \Lambda\}],$$

the closed linear span in $L_p(\mathbb{R})$ of the set $\{f_{(\lambda)} : \lambda \in \Lambda\}$, where $\Lambda \subseteq \mathbb{R}$ is discrete under the assumption that $(f_{(\lambda)})_{\lambda \in \Lambda}$ is basic (under some ordering), unconditional basic, a Schauder frame (under some ordering) or part of a biorthogonal system.

Some results we obtain are

(1) If $(f_{(\lambda)})_{\lambda \in \Lambda}$ is a basis (or Schauder frame) for $X(f, \Lambda) \subseteq L_1(\mathbb{R})$ then $X(f, \Lambda)$ embeds isomorphically into ℓ_1.

(2) If $1 < p \leq 2$ and $(f_{(\lambda)})_{\lambda \in \Lambda}$ is unconditional basic then it is equivalent to the unit vector basis of ℓ_p.

(3) If $2 < p \leq 4$ and $(f_{(\lambda)})_{\lambda \in \Lambda}$ is unconditional basic then $X(f, \Lambda)$ embeds into ℓ_p.

(4) For $4 < p < \infty$ there exists $f \in L_p(\mathbb{R})$ and $\Lambda \subseteq \mathbb{N}$ so that $(f_{(\lambda)})_{\lambda \in \Lambda}$ is unconditional basic and $X(f, \Lambda)$ contains an isomorph of $L_p(\mathbb{R})$.

We report on recent joint work with Th. Schlumprecht, B. Sari and B. Zheng.