Algebraic Combinatorics
Combinatoire algébrique

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Toward a basis of diagonal harmonic alternants

The space of diagonal harmonic alternants is $\mathrm{HA}_n = C\big[E_\lambda \Delta_n[X]\big]$ where Δ_n is the vandermonde determinant, $E_k = \sum y_i \partial_{x_i}$ and $E_\lambda = E_{\lambda_1} E_{\lambda_1} \cdots E_{\lambda_1}$. This space is naturally bigraded by $\binom{n}{2} - |\lambda|$ and $\ell(\lambda)$. It is known that the dimension of HA_n is the Catalan number C_n. In fact even the bi-graded dimension of HA_n is known as the q-t-Catalan number $C_n(q,t)$. Yet, no explicit basis is known for this space.

We construct an explicit basis of certain graded components of HA_n that is valid as long as $n > |\lambda|$.

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Applications of quasi-symmetric functions and noncommutative symmetric functions in permutation enumeration

The descent set of a sequence $a_1 a_2 \cdots a_n$ of integers is the set $\{i \mid a_i > a_{i+1}\}$. It is known that if π and σ are sequences with no elements in common, then the multiset of descent sets of the shuffles of π and σ depends only the descent sets of π and σ. This result gives an algebra of descent sets, which is isomorphic to the algebra of quasi-symmetric functions. The descent number of a sequence is the cardinality of the descent set. The descent number and several other statistics related to descents have the same shuffle-compatibility property as the descent set. They correspond to certain quotients of the algebra of quasi-symmetric functions, and thus to sub-coalgebras of the dual coalgebra of noncommutative symmetric functions.

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Hopf objects between the permutahedra and associahedra

We study the multiplihedra, a relatively new family M of polytopes nestled between the permutahedra P and the associahedra A. The latter families were given interesting Hopf algebra structures by Malvenuto–Reutenauer and Loday–Ronco, respectively. In the work of Aguiar–Sottile, these Hopf structures were largely explained based on geometric properties of P and A (for example, a description of their primitive elements was given in terms of the 1-skeletons of the polytopes). In this talk, we define a structure on M making it a module over P and Hopf module over A. We also use its 1-skeleton to exhibit the fundamental theorem of Hopf modules, giving an explicit basis of coinvariants in M. Time permitting, we indicate a whole zoo of other Hopf objects, yet to be studied, surrounding P, M, and A.

This is joint work with F. Sottile and S. Forcey.

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Posets Isomorphisms in the Hopf Algebra of Tableaux

This work is concerned with some properties of the Malvenuto–Reutenauer Hopf algebra of Young tableaux.

In the course of a recent study of the properties of four partial orders on Young tableaux, Taskin showed that the product of two tableaux of respective size n and m is an interval in each one of four partial orders defined on the set of tableaux of size $n + m$. We are interested in the relations between these intervals, with respect to the weak order on tableaux also called Young tableauhedron.

We want to show that for any quadruple (t_1, t_2, t_3, t_4) of standard Young tableaux such that t_1 and t_3 have the same shape λ while t_2 and t_4 have the same shape μ:

- the intervals describing the products $t_1 \times t_2$ and $t_3 \times t_4$ are isomorphic and the isomorphism between the two intervals preserves the shapes of the tableaux.

And for any couple (t_1, t_2) of standard Young tableaux:

- the intervals describing the non commutative products $t_1 \times t_2$ and $t_2 \times t_1$ are isomorphic and the isomorphism between the two intervals preserves the shapes of the tableaux.