Nonlinear Wave Equations and Applications
Équations d'ondes non linéaires et leurs applications
(Org: **Walter Craig** (McMaster) and/et **Catherine Sulem** (Toronto))

STEPHEN ANCO, Brock University, St. Catharines, Ontario
Symmetry analysis of nonlinear wave equations in $n > 1$ dimensions

Symmetry analysis has several important uses in the study of nonlinear evolution equations, particularly for

(1) identifying critical dimensions,

(2) deriving conserved norms and conservation identities, and

(3) finding explicit solutions with invariance properties.

Applications to semilinear wave equations, Schrodinger equations, and generalized Korteveg–de Vries equations in $n > 1$ dimensions will be presented.

OLIVER DIAZ-ESPINOSA, McMaster University
Long wave expansions for water waves over random bottom

We introduce a technique, based on perturbation theory for Hamiltonian PDEs, to derive the asymptotic equations of the motion of a free surface of a fluid over a rough bottom (one dimension). The rough bottom is described by a realization of a stationary mixing process which varies on short length scales.

We show that the problem in this case does not fully homogenize, and random effects are as important as dispersive and nonlinear phenomena in the scaling regime. We will explain how these technique can be generalized to higher dimensions.

CLEMENT GALLO, McMaster University, 1280 Main Street W., Hamilton, ON L8S 4K1
Transverse instability for the dark solitons of the cubic defocusing NLS equation

In one space dimension, the cubic defocusing Nonlinear Schrödinger equation

$$i\partial_t u + \Delta u + (1 - |u|^2)u = 0, \quad (t, x) \in \mathbb{R} \times \mathbb{R}^d$$

admits solitary waves which do not vanish at infinity, the so-called dark solitons.

These dark solitons are orbitally stable for the dynamic of the one-dimensional equation ($d = 1$).

The dark solitons can also be seen as solutions of the two-dimensional equation ($d = 2$), being constant in the transverse direction.

The purpose of this talk is to show that they are nonlinearly unstable for the dynamic of the two-dimensional equation.

JIANSHENG GENG, McMaster University, Hamilton, Ontario L8S 4K1
Invariant Tori of Full Dimension for a Nonlinear Schrödinger Equation

In this talk, we consider the one-dimensional nonlinear Schrödinger equation

$$\mathrm{i}u_t - u_{xx} + mu + f(|u|^2)u = 0$$

with periodic boundary conditions or Dirichlet boundary conditions, where f is a real analytic function in some neighborhood of the origin satisfying $f(0) = 0$, $f'(0) \neq 0$. We prove that for each given constant potential m, the equation admits a Whitney smooth family of small-amplitude, time almost-periodic solutions with all frequencies. The proof is based on a Birkhoff normal form reduction and an improved version of the KAM theorem. Thus, we give an affirmative answer to an open problem stated in Pöschel (Ergodic Theory Dynam. Systems **22**(2002), 1537–1549) and Bourgain (J. Funct. Anal. **229**(2005), 62–94).

PHILIPPE GUYENNE, Department of Mathematical Sciences, University of Delaware, Newark, DE 19176, USA
Hamiltonian formulation and long wave models for internal waves

We derive a Hamiltonian formulation of the problem of a dynamic free interface (with rigid lid boundary conditions), and of a free interface coupled with a free surface, in view of modeling internal waves in oceans. Based on the linearized equations, we highlight the discrepancies between the cases of rigid lid and free-surface boundary conditions, which in some circumstances can be significant. We also derive systems of nonlinear dispersive long-wave equations in the large-amplitude regime, and numerically compute their solitary wave solutions. Comparisons with other weakly and fully nonlinear results show good agreement.

KONSTANTIN KHANIN, University of Toronto, Department of Mathematics, 40 St. George Street, Toronto, Ontario M5S 2E4
Localization and pinning for directed polymers

We shall present few results (joint with Yu. Bakhtin) on localization for directed polymers. Directed polymers can be considered as random walks in random potential. They play important role in analysis of parabolic Anderson model and random forced Burgers equation.

We are mostly interested in the case when the random potential has the product structure. Namely, it is given by the product of two terms. The first one is a space-dependent potential, while the second is the white noise in time. We show that corresponding polymers are localized provided that the spatial part of the potential has a large maximum (or minimum). We also consider the case when the spatial potential is a stationary process. In this case we show that polymers at zero temperature (action-optimizing paths) has strong pinning properties. We calculate critical exponents for the optimal action fluctuations and for transversal fluctuations of optimal paths. We also show that probability distribution for normalized optimal action fluctuations converges to the universal limit as $t \to \infty$.

DAVID LANNES, McMaster University, Hamilton, Ontario, Canada
The Camassa–Holm and Degasperis–Procesi equations and water waves

The Camassa–Holm and Degasperis–Procesis are well-known bi-Hamiltonian equations with a very rich structure. The aim of this talk is to show how these equations can be seen as model equations in the water wave theory and to point out their relevance to the description of wave breaking phenomena.

JEREMY QUASTEL, University of Toronto
Wiener meets Kortweg and deVries

Gaussian white noise is an invariant measure for KdV on the circle. We explain what this means, and why it is true.

Joint work with Benedek Valko.