A counterexample to Ganea’s conjecture with minimum dimension

In 1967, T. Ganea conjectured that for any finite CW-complex and \( r \geq 1 \) it ought to hold that \( \text{cat}(X \times S^r) = \text{cat} X + 1 \), where \( \text{cat} \) is the Lusternik–Schnirelmann category. This conjecture has been readily disproved by N. Iwase. A 7-dimensional CW-complex \( X \) such that for sufficiently large \( r \), \( \text{cat}(X \times S^r) = \text{cat} X = 2 \) is constructed. Such space \( X \) is then proved to be a minimum dimensional counterexample to Ganea’s conjecture.