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The Isotropic Bound on the Independence Number

If M is a symmetric matrix over some field $\mathbb{F}$ with $2 \neq 0$, then an *M-isotropic subspace* is a subspace U such that

$$u^T M u = 0$$

for all $u \in U$. The maximum dimension of an M-isotropic subspace is called the *Witt index* of M. The *independence number* of a graph G, denoted $\alpha(G)$, is the maximum size of a set of pairwise nonadjacent vertices in G. If $\hat{A}$ is an *$\mathbb{F}$-weight matrix* for G, that is, a matrix with the property that $\hat{A}_{ij} = 0$ when ij is not an edge of G and $\hat{A}_{ij} \in \mathbb{F}$ otherwise, then

$$\alpha(G) \leq \iota_{\mathbb{F}}(\hat{A}) \tag{1}$$

This upper bound is called the *isotropic bound* over $\mathbb{F}$. The isotropic bound is a generalization of a result due to Cvetković which states that

$$\alpha(G) \leq \iota_{\mathbb{R}}(G) = n_0 + \min\{n^+(A), n^-(A)\}$$

where A is the ordinary adjacency matrix of G with $A_{ij} = 1$ if ij is an edge, and $A_{ij} = 0$ otherwise, and n_0, n^+, and n^- denote the numbers of 0, negative, and positive eigenvalues respectively.

It is natural to ask how good the bound (1) can be, and particular for which graphs equality can be attained in the bound.

I will discuss the answers to these questions over various fields and give a classification of the graphs that attain bound over a finite field $\mathbb{F}$ with $2 \neq 0$.