Mathematics of Finance
Finance mathématique

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Risk neutral measures for GARCH option pricing with normal variance-mean mixture examples

Under GARCH models the markets are incomplete so there are an infinite number of risk neutral measures for pricing contingent claims. In general the choice of such a measure has to be justified by an economic argument. This paper investigates risk-neutralized dynamics of various classes of normal variance mean mixture GARCH models using different martingale measures approaches. We discuss the properties of these and we show that for some special cases some of the changes of measure considered lead to similar risk neutralized GARCH dynamics. Numerical examples on pricing European options on S&P 500 index emphasize the importance of the choice of the pricing kernel.

DAVE BOLDER, Bank of Canada

Simple Joint Macroeconomic and Term-Structure Models

The primary objective of this paper is to compare a variety of joint models of the term structure of interest rates and the macroeconomy. To this end, we consider six alternative approaches. Three of these models follow from the work of Diebold and Li (2003) with a generalization in Bolder (2006). The fourth model is a regression-based approach motivated entirely by empirical considerations. The fifth model follows from the seminal work of Ang and Piazzesi (2003), who suggest a joint macro-finance model in a discrete-time affine setting. The final model, which we term an observed-affine model, represents an adjustment to the Ang–Piazzesi model that essentially relaxes restrictions on the state-variable dynamics by making them observable. The observed-affine model is similar in spirit to work by Colin–Dufresne, Goldstein, and Jones (2005) and Cochrane and Piazzesi (2006). Using monthly Canadian data from 1973 to 2005, we compare each of these models in terms of their out-of-sample ability to forecast the transition density of zero-coupon rates. We also examine a simple approach aimed at permitting a subset of the parameters in the non-affine models to vary over time. We find, similar to Bolder (2006), that the Diebold and Li (2003) motivated approaches provide the most appealing modelling alternative across our different comparison criteria.

ABEL CADENILLAS, University of Alberta, Department of Mathematical and Statistical Sciences

Optimal Dividend Policy When There Are Business Cycles

We consider a company whose cash reservoir is affected by macroeconomic conditions. Specifically, we model the cash reservoir as a Brownian motion with drift and volatility modulated by an observable continuous-time Markov process that represents the regime of the economy. The objective of the management is to select the dividend policy that maximizes the expected total discounted dividend payments to be received by the shareholders.

We obtain the analytical solution for this problem. Our solution shows, among other things, that the optimal dividend policy depends strongly on the macroeconomic conditions. The mathematical techniques are those of stochastic control with regime switching.

Joint work with L. R. Sotomayor.

JOE CAMPOLIETI, Wilfrid Laurier, Waterloo, ON
Closed-Form Spectral Expansions for First-Passage Time Densities, Lookback and Barrier Options under New Families of Diffusions

In this talk I will present a general framework for obtaining closed-form spectral expansions for first-passage time (FPT) densities and corresponding transition probability densities for several new families of one-dimensional transformed Markov diffusions with killing. The transformed diffusions are characterized by nonlinear local volatility functions with multiply adjustable parameters and are generated by applying a so-called diffusion canonical transformation method on an underlying solvable diffusion with killing. Based on the transition and FPT densities, analytically closed-form (and rapidly convergent) spectral expansions are also obtained for lookback and (single and double) barrier options for such transformed processes. The approach is presented for a general class of diffusions and leads to several applications in finance. Specifically, this talk will discuss some analytically exact results for three new main families of diffusion models. The first-hitting time densities and option pricing formulas recover the recently derived expansions in the literature for the simpler processes that include the CEV and many other diffusion models as special cases of our new formulas.

MATT DAVISON, University of Western Ontario, London, ON
A model of contagion in the P&C Insurance industry

The Canadian Property and Casualty Insurance industry is a collection of about 200 firms, none of whom controls more than 10% of the market. In order to safeguard the customers of this industry from harm, the Property and Casualty Insurance Compensation Corporation (PACICC) has been formed. In the unlikely event that an insurer becomes insolvent, PACICC pays any outstanding claims to that insurer as well as refunding unearned insurance premia. The funds for this are obtained by an after-the-fact levy on the surviving members. This levy leaves the industry at risk of contagion—healthy companies being forced into insolvency to pay this levy.

This talk presents the results of a data-intensive study done, in conjunction with PACICC, to investigate this contagion risk.

This work is joint with Sharon Wang, Darrell Leadbetter and Lindsay Anderson.

KELDON DRUDGE, Prism Valuation and University of Waterloo
Measuring gap risk for CPPI strategies: implied and otherwise

Constant Proportion Portfolio Insurance (CPPI) is a an investment strategy designed to provide principal protection along with the potential of upside in the case of performance of the underlying asset. In continuous time, with continuous rebalancing (and no jumps in the underlying asset value) the strategy is guaranteed to end with a portfolio worth at least the initial investment amount. Therefore the main risk in implementing the strategy in practice is "gap risk", the probability that the asset price drops more quickly than the portfolio can be rebalanced. We outline some attempts to calibrate a simple jump diffusion model to provide an estimate of the cost of this gap risk for CPPI strategies.

MARCOS ESCOBAR-ANEL, Ryerson University, 350 Victoria Street, Toronto, Ontario M5B 2K3
Stochastic Correlation in the Valuation of Low-Dimensional Derivative Contracts

In this talk, several methods of valuation of low-dimensional derivatives, like spread options, are considered. Special emphasis is given to methods based on numerical solutions using recombining trees and/or characteristic functions. A more realistic approach towards dependence structure is achieved by working with continuous time processes that capture stylized facts as stochastic correlation among stocks and volatilities. Standard constant correlation tree methods are enriched with the notion of stochastic correlation, leading to recombining trees that show fast rate of convergence and are easy to implement. The sensitivity and hedging analyses with respect to the stochastic correlation parameters are performed, showing that a constant correlation model systematically overprices both the spread option value as well as the hedging parameters.

MATHEUS GRASSELLI, McMaster University
Insurance products in markets with stochastic volatility

We consider the problem of partially hedging the risk of an insurance contract by trading in a financial market. Because of the typical duration of such contracts, it is unrealistic to assume that the volatilities of the underlying financial assets are constant. Accordingly, we investigate how the results of Young and Zariphopoulou (2002) and Young (2003) can be generalized for markets with stochastic volatility. We show that stochastic volatility does not affect the indifference price of either single-life insurance and pure-endowment contracts, unless the agent uses a random horizon, in which case the indifference price depends on the speed of mean reversion and long term average of the volatility, as well as the mean rate of return of the stock. Equity-index contracts, on the other hand, exhibit a much richer dependence on volatility, which we explore numerically using the perturbation technique of Papanicolau, Fouque and Sircar (2000).

JEREMY GRAVELINE, University of Minnesota
Exchange Rate Volatility and the Forward Premium Anomaly

I ask whether the forward premium anomaly is consistent with an arbitrage-free model for the exchange rate and term structures of interest rates in two currencies. I use a two-currency term structure model to examine two sets of currency pairs: the US Dollar and British Pound, and the US Dollar and Euro. Previous papers in this literature have failed to match exchange rate volatility, which is an essential component of the risk premium in exchange rate returns. A distinguishing feature of this paper is that I use exchange rate option prices to estimate the model. Options provide valuable information because they are sensitive to the level of volatility and to the pricing of volatility risk. When options are used to estimate the model, it successfully captures both exchange rate volatility and the term structure of interest rates in both currencies. Using simulated data, I show that the model also replicates the empirical findings in Fama (1984) and is consistent with the forward premium anomaly.

TOM HURD, McMaster University
First passage problems for jump diffusions arising in finance

Many aspects of pricing and hedging in option theory and credit risk boil down to problems of computing the first passage distribution of underlying stochastic processes. Techniques are well developed in the case of diffusion models, and connect with deep results of probability theory. For various reasons, first passage problems become more difficult, yet still very interesting, when the underlying processes have jumps. In this talk I will survey some areas of financial mathematics where first passage problems arise, and show various mathematical techniques for dealing with them. In the end, I will introduce an apparently new method applicable in a very useful class of models involving time changed Brownian motions.

CODY HYNDMAN, Concordia University, 1455 boul. de Maisonneuve O., Montreal, Quebec H3G 1M8
Forward-backward stochastic differential equations and term structure derivatives

We consider the application of forward-backward stochastic differential equations (FBSDEs) to the problem of pricing and hedging various term structure derivatives. The underlying short-rate model considered is a multi-factor affine term structure model. Characterizing the underlying dynamics and derivative prices as FBSDEs allows for the implementation of simulation-based numerical methods for solving FBSDEs.

SEBASTIAN JAIMUNGAL, University of Toronto
From Spot to Forward Stochastic Volatility Models for Commodities

It is well known that stochastic volatility is an essential feature of commodity prices. By using methods of singular perturbation theory, I will show how to obtain approximate but explicit closed form pricing equations for forward contracts and options

on single- and two-name forward prices. Both spot and forward price commodity models, based on a fast mean-reverting stochastic volatility driving factor, will be explored. For spot price models the single factor mean-reverting spot model as well as a two-factor generalization, in which the long-run mean is itself mean-reverting, are extended to include stochastic volatility. For forward price models, I will adopt an HJM-like framework with stochastic volatility extensions and include an unspanned volatility source. The various approximation formulas produce realistic implied volatility smiles and are useful calibration and pricing tools.

MADHU KALIMIPALLI, School of Business Economics, Wilfrid Laurier University
Information Content of Equity Volatility for Default and Liquidity Risks in Corporate Bond Market

Idiosyncratic equity volatility (idiosyncratic volatility, hereafter) refers to the firm-specific risk or volatility adjusted for the portfolio and market-wide effects. It represents the diversifiable component of risk from holding a stock. In this paper, we examine the specific channels through which the idiosyncratic volatility affects the corporate bond spreads. We study how idiosyncratic volatility affects different components of bond spreads. We address the following questions. Does a higher idiosyncratic volatility imply a higher default risk or liquidity risk? What is the effect of idiosyncratic volatility on bond spreads once we control for

(a) default risk? and

(b) both default and liquidity risks?

What is the relative impact on default and liquidity risk components of the bond spread, for movements in idiosyncratic volatility. By exploring these questions, we hope to better understand the relationship between the credit and liquidity components of corporate bond spreads, and the underlying firm specific news events.

VALERY KHOLODNYI, Platts Analytics
The Semilinear Evolution Equation for American Contingent Claims: Successive Approximations and Bounds

We present the semilinear evolution equation for American contingent claims in the entire domain of the state variables introduced earlier by the author. The nonlinear term in this equation can be financially interpreted as a cash flow that should be received to compensate for the losses due to holding an American contingent claim unexercised in the exercise region.

With the help of the method of variation of constants the semilinear evolution equation for American contingent claims can be represented as a nonlinear integral equation also introduced earlier by the author. This integral equation can be financially interpreted as the early exercise premium representation of the value of an American contingent claim, that is as the sum of the value of the corresponding European contingent claim and the early exercise premium.

We show that although the successive approximations do not, in general, converge to the exact solution of the integral equation, for a suitable initial approximation such as the value of the corresponding European contingent claim, the successive approximations can provide relatively simple approximations as well as lower and upper bounds for the exact solution, that is for the value of an American contingent claim.

ADAM KOLKIEWICZ, University of Waterloo, Department of Statistics and Actuarial Science, Waterloo, Ontario
Estimation for Diffusion Processes Using Reverse-Time Specifications

In the paper we consider methods of estimation for diffusion processes based on explicit specifications of the dynamic of the reversed process. In the parametric setting we show the equivalence of martingale estimating equations derived from the forward and the backward dynamics of the process. For nonparametric methods of estimation of the drift parameter, we justify using the forward and the reversed trajectories jointly and present an argument suggesting that such an approach should lead

to a significant gain in efficiency. We demonstrate that this is indeed the case in the context of the Cox, Ingersoll, and Ross model of short interest rate. Depending on the parameter set and the criterion, the reduction of the integrated errors ranges from 100% up to 400%.

HUA LI, University of Calgary
Pricing and hedging European Options with uncertain parameters

In recent years, fuzzy set theory has been introduced as a means of modeling the uncertainties of the input parameters of the Black–Scholes European options pricing formula. However, some standard assumptions underlying the Black–Scholes model including those of constant interest rate and volatility no longer hold in fuzzy environments. Therefore, it is inappropriate to price options with uncertain parameters based on the Black–Scholes formula.

In this talk, we propose a methodology for option pricing under fuzzy environments which is essentially different from the Black–Scholes option pricing framework. We build a nonlinear fuzzy-parameter PDE model for obtaining the fuzzy option prices and we develop dominating optimal hedging strategies which provide valuable insights for risk management and trading in financial markets.

ROMAN MAKAROV, Wilfrid Laurier University, 75 University Avenue West, Waterloo, Ontario, Canada
Analysis and Classification of Nonlinear Diffusion Financial Models

We present a new approach for analyzing probabilistic properties of one-dimensional time-homogeneous diffusions that are characterized by drift, $\lambda(x)$, and diffusion, $\nu(x)$, coefficient functions. In particular, we analyze whether or not a diffusion process can admit solutions that preserves the drift rate, i.e., for which $\frac{d}{dT}\mathsf{E}[X_T \mid X_t] = \mathsf{E}[\lambda(X_t)]$, $t \leq T$, holds. This property can be viewed as a generalization of the martingale property of driftless processes, when $\mathsf{E}[X_T \mid X_t] = X_t$ and, therefore, $\frac{d}{dT}\mathsf{E}[X_T \mid X_t] = 0$ hold. Our approach is based on classical Green's functions methods for generally singular second order Sturm–Liouville ODEs that arise from the Laplace transform of the Kolmogorov PDE. By employing the Liouville–Green approximation and asymptotic analysis of the fundamental solutions to the corresponding second order ODE, we investigate the qualitative behaviour of the probability density solutions in the neighborhoods of the endpoints of the processes. In particular, we apply our analysis to quite general one-dimensional processes whose drift and volatility functions are either power series expansions or asymptotic equivalents of power functions in neighborhoods of the endpoints of the state domain. In doing so, we arrive at a complete "exponent classification" of such processes with respect to their underlying probabilistic properties. The results obtained are illustrated with various models arisen in mathematical finance.

MARIANITO RODRIGO, Instituto Tecnológico Autónomo de México
A new representation of the local volatility surface

We address the problem of recovering the local volatility surface from market option prices. An Ansatz approach is employed to obtain a semi-explicit solution of Dupire's forward equation. This solution, in turn, gives rise to a new expression for the volatility surface in terms of the price of a European call or put. We also show the results of a numerical simulation.

Joint work with R. Mamon, University of Western Ontario.

DAVE SAUNDERS, University of Waterloo, 200 University Avenue West, Waterloo, Ontario N2L 3G1
Risk Contributions of Systematic Factors in Multi-Factor Credit Risk Models

Multi-factor credit portfolio models are used widely today for measuring and managing economic capital as well as for pricing credit portfolio instruments such as collateralized debt obligations (CDOs). Commonly, practitioners allocate capital to the portfolio components, such as individual sub-portfolios, counterparties, or transactions. The hedging of credit risk is generally

also focused on the "deltas" of the underlying names in the portfolio. Understanding the contribution to economic capital or pricing of the systematic factors (or credit drivers), which are at the heart of a multi-factor credit model, can lead to better methodologies for managing concentration risk and hedging credit portfolios effectively.

However, since the total capital is not a homogeneous function of these factors, the standard theory of marginal capital contributions does not work well. We present and compare several methodologies for defining and measuring the risk contributions of systematic credit factors.

LUIS SECO, Toronto

ANATOLIY SWISHCHUK, University of Calgary, 2500 University Drive NW, Calgary, Alberta T2N 1N4
Pricing Variance Swaps for Stochastic Volatilities with Delay and Jumps

The valuation of the variance swaps for stochastic volatility with delay and jumps is discussed in this talk. We provide some analytical closed forms for the expectation of the realized variance for the stochastic volatility with delay and jumps. The jump part in our model is finally represented by a general version of compound Poisson processes. As applications of our analytical solutions, a numerical example using S&P60 Canada Index (1998–2002) is then provided to price variance swaps with delay and jumps. Finally, we find that this model not only keeps some good features of the previous model without jumps but is also easy and quick to implement.

This is a joint talk with Li Xu (Department of Mathematics and Statistics, University of Calgary, Calgary, Alberta).

KEN VETZAL, Centre for Advanced Studies in Finance, University of Waterloo, Waterloo, ON N2L 3G1
Earnings Volatility and Corporate Bond Spreads

In general terms, structural models of risky debt share the common feature of treating corporate debt as a call option written on the firm's assets. In other words, equity holders are entitled to repurchase the firm's assets from debt holders by fully repaying the interest and par value specified in the debt contract. This modelling tradition began with Merton (1974), who considered only the simplest case of a zero-coupon bond with no bankruptcy costs. More recent papers have considered more complex and realistic situations. A variety of empirical tests (see, e.g., Eom, Helwege, and Huang (2004)) generally indicate that structural models underpredict corporate bond spreads, which is partially attributable to estimates of asset return volatility.

Rather than modelling asset return volatility directly, we follow more recent authors such as Hackbarth, Miao and Morellec (2006) who adopt a more detailed perspective, beginning with the issue of how asset values are determined by corporate earnings. At a simple intuitive level, higher earnings volatility increases the probability of default, thus raising bond yields. We describe a regime-switching model in which the level of earnings volatility changes across the various states and explore the implications of this for corporate bond spreads. We also conduct an empirical examination of the effects of earnings volatility on bond yields using corporate bond trading data. We find that earnings volatility provides significant additional explanatory power for the cross-section of corporate bond yields.

XIKUI WANG, University of Manitoba, Department of Statistics, Winnipeg, Manitoba R3T 2N2
Sequential optimization under uncertainty

Two kinds of problems are discussed to illustrate the use of statistics in finance and economics. For the optimal portfolio problem, the distribution of investment returns is assumed to be unknown and the Bayesian approach is applied. The sequential portfolio model under uncertainty is formulated as a Markov decision process and the optimal strategy is characterized. For the problem of dynamically pricing a product under uncertainty, we assume that the demand function is determined by a compound

Poisson process with unknown parameters. The optimal pricing problem is formulated as a bandit model and the key issue is to compromise between information gathering (so as to reduce uncertainty and make better informed decisions in the future) and immediate payoff (so as to achieve a certain measure of economic revenues). The optimal strategy and its properties are addressed.