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Maximal Projective Codes

For $n \geq k$, an $(n, k, d)_q$ -code C is a collection of q^k n -tuples (or *codewords*) over an alphabet \mathcal{A} of size q such that the minimum (Hamming) distance between any two codewords of C is d . For such a code, the Singleton bound ($|C| \leq |\mathcal{A}|^{n-d+1}$) gives $d \leq n - k + 1$. The *Singleton defect* of C , $S(C)$, is defined by $S(C) = n - k + 1 - d$. A code C' obtained by deleting some fixed coordinate from each codeword of C is called a *punctured code* of C . In the case that $S(C') = S(C)$, C is said to be an *extension* of C' , equivalently, C' is said to be *extendable* to the code C . A code is *maximal* if it admits no extensions.

In the special case that $\mathcal{A} = GF(q)$ and C is a vector space of dimension k , C is a *linear* $(n, k, d)_q$ -code. C then has an associated generator matrix G whose columns can be considered as a projective multiset \mathcal{G} of n points in $PG(k-1, q)$ at most $n-d$ per hyperplane—called a *projective system* associated with C . If the points in \mathcal{G} are distinct (so that essentially there are no repeated coordinates), C is a *projective code*. Hence, complete (n, r) -arcs in $PG(k-1, q)$ and projective $(n, k, n-r)_q$ -codes that admit no projective extensions are equivalent objects. This begs the question: *Is a projective code corresponding to a complete arc necessarily maximal?* We show that projective codes of reasonable length admit only projective extensions. Many examples of large complete arcs exist; our results show that in many cases the corresponding codes are maximal. The methods used are based on the Bruen–Silverman model of linear codes utilizing coprimitive sets as well as the theory of Rédei blocking sets.

Joint work with A. A. Bruen.