Calculus of Variations in Physics, Geometry and Economics
Calcul des variations, géométrie et économie
(Org: **Robert McCann** and/et **Benjamin Stephens** (Toronto))

ALMUT BURCHARD, University of Toronto, Department of Mathematics, 40 St. George Street, Toronto, ON M5S 2E4
Solitary waves on an elastic curve

I will discuss the stability of solitary waves on an infinite elastic curve moving in three-dimensional space. While slow-moving waves are unstable, higher wave speeds stabilize the solitary wave; I will try to explain the mechanism.

This is joint work with Erin Valenti.

ADRIAN BUTSCHER, Stanford University, Stanford, CA
Collapsing Constant Mean Curvature Surfaces in Riemannian Manifolds

I will discuss the construction of CMC surfaces by gluing techniques in general Riemannian manifolds. These are found by assembling an approximate solution consisting of small geodesic spheres connected by embedded catenoids, which can then be perturbed to an actual solution under certain conditions on the geometry of the approximate solution. The resulting surfaces have very large mean curvature. An interesting phenomenon is that in the case where the ration of the norm of the second fundamental form to the size of the mean curvature is very large, then it is possible to have solutions which exhibit behaviour that is very different from the 'classical' behaviour (e.g. no one-ended CMC surfaces, CMC surfaces are cylindrically bounded) that occurs in Euclidean space.

ALBERT CHAU, Department of Mathematics, University of British Columbia, Room 121, 1984 Mathematics Road, Vancouver, BC V6T 1Z2
The Kahler–Ricci flow on complete non-compact manifolds and uniformization

Introduced in 1982 by Richard Hamilton, the Ricci flow is one of the most important equations in differential geometry. It is a geometric evolution equation providing a powerful analytic tool used to deform a Riemannian metric on a Riemannian manifold. The Ricci flow has found fundamental application in topology, Riemannian geometry and complex/Kahler geometry. In this talk I will discuss the Kahler–Ricci flow on complete non-compact Kahler manifolds. I will then discuss the application of the Kahler–Ricci flow to the uniformization of complete non-compact Kahler manifolds and to Yau's uniformization conjecture. Yau's conjecture states: a complete non-compact Kahler manifold with positive holomorphic bisectional curvature is biholomorphic to complex Euclidean space.

ALESSIO FIGALLI, University of Nice–Sophia Antipolis, Labo. J.-A. Dieudonné, UMR CNRS 6621, Parc Valrose, 06108 Nice Cedex 02, France
A mass transportation approach to quantitative isoperimetric inequalities

In this talk I will show how one can prove a sharp quantitative version of the anisotropic isoperimetric inequality by exploiting mass transportation theory, especially Gromov's proof of the isoperimetric inequality and the Brenier–McCann Theorem.

This is a joint work with F. Maggi and A. Pratelli.

YUXIN GE, U. Washington and U. Paris 12
Regularity of optimal transportation maps on nearly spherical manifolds

Given a couple of smooth positive measures of same total mass on a compact Riemannian manifold M, we look for a smooth optimal transportation map G, pushing one measure to the other at a least total squared distance cost. The recent local C^2 estimate of Ma–Trudinger–Wang enabled G. Loeper to treat the standard sphere case. In this talk, we discuss this topic on manifolds with curvature sufficiently close to 1 in C^2 norm.

This is a joint work with P. Delanoe.

MARC HENRY, Université de Montréal
Mass transportation duality in econometrics

A general framework is given to analyze the falsifiability of economic models based on a sample of their observable components. It is shown that, when the restrictions implied by the economic model are insufficient to identify the unknown quantities of the model, the duality of optimal transportation with zero-one cost function delivers interpretable and operational formulations of the hypothesis of model correctness from which tests can be constructed to falsify the model.

YOUNG-HEON KIM, University of Toronto, Toronto, ON, Canada
Curvature and continuity of optimal transport

We will discuss continuity of optimal transport maps, in view of a pseudo-Riemannian structure which we have formulated recently. A necessary condition for the continuity is given as some non-negativity condition on the curvature of this pseudo-Riemannian metric. This result gives a natural geometric frame work and new perspectives for the regularity theory of Ma, Trudinger, Wang and Loeper on the potential functions of optimal transport; it also yields some extensions of previous results and new examples.

This is joint work with Robert McCann (University of Toronto).

ABDESLEM LYAGHFOURI, Fields Institute, Toronto, Canada
Hölder Continuity of Solutions to the A-Laplace Equation Involving Measures

In joint work with Samia Challal, we show an optimal Hölder continuity for bounded solutions of the equation $-\Delta_A u = \mu$ provided that $\mu\big(B_r(x)\big) \leqslant Cr^{n-1}$ for any ball $B_r(x) \subset \Omega$. The A-Laplace operator is defined by $\Delta_A u = \mathrm{div}\big(\frac{a(|\nabla u|)}{|\nabla u|}\nabla u\big)$, where $A(t) = \int_0^t a(s)\,ds$, a is an increasing C^1 function from $[0,+\infty)$ into $[0,+\infty)$ which satisfies $a(0) = 0$ and

$$a_0 \leqslant \frac{ta'(t)}{a(t)} \leqslant a_1 \quad \forall t > 0, \quad a_0, a_1 \text{ positive constants.}$$

ABBAS MOAMENI, Queens' University
Existence and concentration of solitary waves for a class of quasilinear Schrödinger Equations

In this talk we prove the existence and qualitative properties of positive bound state solutions for a class of quasilinear Schrödinger equations in dimension $N \geq 3$. We rely on a penalization technique, in a nonstandard Orlicz space context, to build up a one parameter family of classical solutions which have finite energy and exhibit, as the parameter goes to zero, a concentrating behavior around some point which we localize.

SANTIAGO MORENO, UBC 121-1984 Mathematics Road, Vancouver, BC V6T 1Z2
Risk Minimization and Optimal Derivative Design in a Principal Agent Game

We study a Principal-Agent model of optimal derivative design where the agents' preferences are of mean-variance type and their types characterize their risk aversion. The set of contracts traded expose the principal to additional risk as measures by a convex risk measure in exchange for a known revenue.

The principal's aim is to minimize her risk exposure by trading with the agents subject to the standard incentive compatibility and individual rationality conditions on the agents' choices. In order to prove that the principal's risk minimization problem has a solution, we first follow the seminal idea of Rochet and Choné and characterize incentive compatible catalogues in terms of U-convex functions. When the impact of a single trade on the principal's revenues is linear as in the recent paper by Carlier, Ekeland and Touzi, the link between incentive compatibility and U-convexity is key to establish the existence of an optimal solution. In our model the impact is non-linear as a single trade has a non-linear impact on the principal's risk assessment. Due to this non-linearity we face a non-standard variational problem where the objective cannot be written as the integral of a given Lagrangian. Instead, our problem can be decomposed into a standard variational part representing the aggregate income of the principal, plus the minimization of the principal's risk evaluation, which depends on the aggregate of the derivatives traded.

DANIEL OFFIN, Queen's University, Kingston, Canada
Stability of minimizing solutions in the N-body problem

We study subsystems of the N-body problem, constructing minimizing noncollision periodic orbits using a symmetric variational method with a finite order symmetry group. The solution of the variational problem gives existence of periodic orbits which realize certain symbolic sequences of rotations and oscillations for any choice of the mass ratio.

The Maslov index of the periodic orbits is then investigated and used to prove the main result which states that the minimimizing curves in the three dimensional reduced energy momentum surface are naturally extended to periodic integral curves which are generically hyperbolic.

ALEXANDER SHNIRELMAN, Concordia University, Montreal
Variational problem in a partially ordered set and the problems of fluid dynamics

We consider 2-dimensional flows of ideal incompressible fluid, i.e., vector fields in a 2-d domain which are divergence-free and tangent to the boundary. The flows have an intrinsic partial order. Minimal elements of this order are stationary and stable solutions of the Euler equations. This is a non-classical variational principle which may be regarded as dual to the Arnold variational principle for stationary flows, while having very different meaning.

The talk is devoted to the connections of this principle and the problem of the long time asymptotics of solutions of the Euler equations, and the numerical solution of this nonclassical variational problem.

ALINA STANCU, Concordia University
On some new characterizations of ellipsoids

Perhaps the most known characterization of ellipsoids defines them as the convex bodies maximizing the affine isoperimetric ratio. In this talk, we will present some new characterizations involving floating bodies and, respectively, illumination bodies.

DENNIS THE, Dept. of Mathematics & Statistics, McGill University, 805 Sherbrooke St. West, Montreal, QC H3A 2K6
The Principle of Symmetric Criticality in Gauge Theory

With respect to the action of a symmetry group G, the principle of symmetric criticality (PSC) roughly states that "critical symmetric points" are "symmetric critical points". PSC is well known to hold if G is compact. After reviewing its formulation due to Palais and more recently Anderson, Fels, and Torre, we:

(1) establish that PSC holds if the orbits are Riemannian symmetric spaces, and

(2) discuss PSC in the context of G-invariant Lagrangians defined on the bundle of connections over homogeneous spaces G/K.

In particular, we examine the non-reductive pseudo-Riemannian homogeneous spaces of dimension 4 recently classified by Fels and Renner (2006). These provide a class of examples where PSC generally fails. There is one interesting exception in this class where PSC holds—in this case, there is a unique G-invariant connection which is "universal" in the sense that it is necessarily a solution of the Euler–Lagrange equations of *any* G-invariant Lagrangian defined on the bundle of connections (in particular, the Yang–Mills Lagrangian).

MAXIM TROKHIMTCHOUK, University of California at Berkeley
Everywhere regularity of certain nonlinear systems

I will talk about nonlinear parabolic systems that are generalizations of scalar diffusion equations. More precisely, I consider systems of the form

$$\mathbf{u}_t - \Delta[\nabla\Phi(\mathbf{u})] = 0,$$

where $\Phi(z)$ is a strictly convex function. I will show that when Φ is a function only of the norm of $\mathbf{u}$, then bounded weak solutions of these parabolic systems are everywhere Hölder continuous and thus everywhere smooth. I will also show that the method used to prove this result can be easily adopted to simplify the proof of the result due to Wiegner on everywhere regularity of bounded weak solutions of strongly coupled parabolic systems.