MARTA BUNGE, McGill University, Department of Mathemaics and Statistics, 805 Sherbrooke Street West, Montreal, QC H3A 2K6

Fundamental Pushout Toposes

The expression 'fundamental pushout' was employed informally in [1] in the special case of a locally connected topos $\mathcal{E}$ bounded over a base topos $\mathcal{S}$, relative to a cover U in $\mathcal{E}$. For a general topos $\mathcal{E}$ bounded over a base topos $\mathcal{S}$, we introduce here three notions which, in the locally connected case, agree. These notions, of decreasing generality, we call respectively 'locally split pushout', 'fundamental pushout' and 'Galois pushout'.

A generalization of the locally connected case is obtained in [?] by resorting to the notion of a locally constant object [?], which makes no reference to connected components. It follows that not every locally split pushout is fundamental. A condition is imposed in [?] to repair this situation but, in the process, even the locally split pushout is lost.

An alternative generalization of the locally connected case arises from the comprehensive factorization of [?]. This gives a localic and not necessarily discrete version of the fundamental pushout. This 'defect' can be corrected by imposing an assumption ('locally quasiconnected') on a Grothendieck topos $\mathcal{E}$, which states that for every object (cover) U in $\mathcal{E}$, the locale of quasicomponents of $\mathcal{E}$/U is spatial and non-trivial. The resulting fundamental pushout need not be Galois in general. In the manner of [?], one can then isolate the 'locally simply quasiconnected' toposes and derive a Galois theory.

References

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