Combinatorics and its Applications to Mathematical Physics
Combinatoires et ses applications en physique mathématique

(Org: **Michael Gekhtman** (Notre Dame) and/et **Michael Shapiro** (Michigan State))

RENZO CAVALIERI, University of Michigan, 530 Church Street, Ann Arbor, MI, USA

Hyperelliptic Hodge integrals

We will discuss the structure of hyperelliptic Hodge integrals (integrals of λ and ψ classes on the hyperelliptic locus of curves). We start by giving a purely combinatorial proof of Faber–Pandharipande's well known formula for $\lambda_g\lambda_{g-1}$, and proceed from there to describe several nice combinatorial features/formulas of more general classes integrals.

LEONID CHEKHOV, Michigan State Univ., East Lansing, USA

Teichmuller theory of bordered surfaces

We propose the graph description of Teichmüller theory of surfaces with marked points on boundary components (bordered surfaces). Introducing new parameters, we formulate this theory in terms of hyperbolic geometry. We can then describe both classical and quantum theories having the proper number of Thurston variables (foliation-shear coordinates), mapping-class group invariance (both classical and quantum), Poisson and quantum algebra of geodesic functions, and classical and quantum braid-group relations. These new algebras can be defined on the double of the corresponding graph related (in a novel way) to a double of the Riemann surface (which is a Riemann surface with holes, not a smooth Riemann surface). We enlarge the mapping class group allowing transformations relating different Teichmüller spaces of bordered surfaces of the same genus, same number of boundary components, and same total number of marked points but with arbitrary distributions of marked points among the boundary components. We describe the classical and quantum algebras and braid group relations for particular sets of geodesic functions corresponding to A_n and D_n algebras.

SHAUN FALLAT, University of Regina

The combinatorics of totally positive minors and implications

A matrix is called totally positive if all of its minors are positive. Recently, such minors have been expressed in terms of parameters associated with a corresponding bidiagonal factorization, and consequently, these minors can be written as subtraction free-expressions in these parameters. This new representation has been recast in a number of different settings, and it continues to arise as a useful tool involving totally positive matrices.

In this talk, I will review this combinatorial representation and discuss some of the interesting resulting applications, including determinantal inequalities and an implication to Jordan canonical forms.

ANNA FELIKSON, Independent University of Moscow, B. Vlassievskii 11, 119002, Moscow, Russia

Combinatorics of Coxeter polytopes

We discuss combinatorial properties of Coxeter polytopes in terms of their missing faces. This leads to a set of simplicial complexes which is useful to differ combinatorial types of simple polytopes. We describe properties of this set and state some conjectures.

SERGEI FOMIN, University of Michigan, Ann Arbor, MI 48109, USA

Cluster algebras associated with bordered surfaces

This is a report on an ongoing joint project with Michael Shapiro and Dylan Thurston devoted to the study of cluster-algebraic structures in decorated Teichmuller spaces of bordered Riemann surfaces with marked points.

CHRISTOF GEISS, Instituto de Matemáticas, Universidad Nacional Autónoma de México, 04510 México DF, México

Cluster algebras arising from preinjective modules

Let Q be a quiver without oriented cycles, $\mathfrak{n}$ the positive part of the symmetric Kac–Moody algebra of type Q and $\Lambda = \mathbb{C}\overline{Q}/\langle\sum_{a\in Q}[a,\overline{a}]\rangle$ the corresponding preprojective algebra. Let $M = \bigoplus_{i=1}^{r} M_i$ be a terminal representation of Q, i.e., the summands of M are a family of indecomposable preinjective representations, closed under successors. Then the full subcategory category

$$\mathcal{C}_M = \{X \in \Lambda\text{-mod}_0 \text{ with } X|_Q \in \mathrm{Add}(M)\}$$

is a Frobenius category and its stable category is 2-Calabi–Yau. Moreover it has a canonical cluster tilting object. We can describe $\mathcal{C}_M$ conveniently as the Δ-good modules of a quasi-hereditary algebra, in this setting, mutation can be described via Δ-dimension vectors.

We have a cluster character ϕ from $\Lambda\text{-mod}_0$ to the commutative ring $U(\mathfrak{n})^*$ which is compatible with our subcategories, so we get a cluster algebra structure on the "image" of $\mathcal{C}_M$. These cluster algebras are closely related to the coordinate rings of certain reduced double Bruhat cells.

This is a report on joint work with B. Leclerc and J. Schröer.

IAN GOULDEN, University of Waterloo, Waterloo, Ontario N2L 3G1, Canada

New combinatorial solutions to the KP equations

We give a new general solution to the KP equations, that specializes to Okounkov's double Hurwitz series, and also has other specializations of interest to combinatorialists.

DAVID JACKSON, University of Waterloo, Waterloo, Ontario, Canada

The ϕ^4-and Penner models of 2d Quantum gravity, the moduli space of curves and properties of 2-cell embeddings of graphs in Riemann surfaces

A *map* is a 2-cell embedding of a graph in a Riemann surface. The generating series for a class of maps is called the *map series* for the class. I shall discuss two questions, one from mathematical physics and the other from algebraic geometry, where map theory reveals the presence of deeper structure and connexions between the two.

I) The ϕ^4-model and $\log(1-\phi)^{-1}$-model (due to Penner) are early models of topological quantum field theory. The relationship between the partition functions for these two models may be explained as a consequence of a functional relationship between two classes of maps in orientable surfaces, one in which all vertices have degree 4 and the other in which there is no restriction on vertex degrees. Moreover, comparable relationships hold for other classes of maps and there is evidence of a natural bijection accounting for these relationships.

II) The generating series for the virtual Euler characteristics for the moduli spaces of complex and for real algebraic curves, respectively, may be shown to be specialisations of the map series for all surfaces through an algebraic parameter associated with Jack symmetric functions. This parameter is conjectured to have an interpretation as an invariant of maps, which then opens the possibility of passing it through the Strebel derivative construction used by Harer and Zagier, to the level of the moduli spaces.

In this talk I shall show how these conjectures arose in the first place.

PETER MAGYAR, Michigan State University, East Lansing, Michigan
Fusion of affine Schubert varieties

We give a unified explanation of various factorization phenomena involving loop groups, affine Grassmannians, and representations of affine Lie algebras. Our main tool is a geometric fusion product analogous to the Feigin–Loktev fusion of affine representations.

BRUCE SAGAN, Michigan State University, East Lansing, MI 48824, USA
Monomial Bases for NBC Complexes

Let G be a graph whose edge set E has been totally ordered. Consider the corresponding NBC complex Δ consisting of all subsets of E which do not contain a broken circuit with respect to the ordering. Let R be the Stanley–Reisner ring of Δ. Jason Brown gave an explicit description of a homogeneous system of parameters for R in terms of fundamental edge-cuts in G. So R modulo this h.s.o.p. is a finite dimensional vector space. We conjecture an explicit monomial basis for this vector space in terms of the circuts of G and prove that the conjecture is true for several families of graphs.

MARK SKANDERA, Lehigh University, Bethlehem, PA 18015
On the cluster basis of $z[x_{1,1},\dots,x_{3,3}]$

We show that the set of cluster monomials for the cluster algebra of type D_4 forms a basis of the $\mathbb{Z}$-module $\mathbb{Z}[x_{1,1},\dots,x_{3,3}]$. We also show that the transition matrices relating the cluster basis of this module to the natural and the dual canonical bases are unitriangular and nonnegative. These results support a conjecture of Fomin and Zelevinsky on the equality of the cluster and dual canonical bases of $\mathbb{Z}[x_{1,1},\dots,x_{3,3}]$. In the event that this conjectured equality is true, our results also imply an explicit factorization of each dual canonical basis element of the module as a product of cluster variables.

JOHN STEMBRIDGE, University of Michigan, Ann Arbor, MI 48109, USA
Admissible W-graphs

Given a Coxeter group W, a W-graph is a combinatorial structure that encodes a W-module, or more generally, a module for the associated Iwahori–Hecke algebra. By a theorem of Gyoja, it is known that every irreducible representation of a finite Coxeter group may be realized as a W-graph. Of special interest are the W-graphs that encode the Kazhdan–Lusztig cell representations of Hecke algebras, and more generally, the cell representations associated to blocks of irreducible representations of real Lie groups.

In this talk, our goal is to isolate some basic features of the W-graphs of cell representations, and use these to create a class of "admissible" W-graphs that is amenable to combinatorial analysis and (we hope) classification. In this direction, we will describe two theorems. First, a Dynkin diagram classification of all rank 2 admissible W-cells, and second, in the simply-laced case, a combinatorial characterization of all admissible W-graphs.

RAVI VAKIL, Stanford University, Stanford, CA 94305, USA
A natural smooth compactification of the space of elliptic curves in projective space

The space of smooth genus 0 curves in projective space has a natural smooth compactification: the moduli space of stable maps, which may be seen as the generalization of the classical space of complete conics. It has a beautiful combinatorial structure. In arbitrary genus, no such natural smooth model is expected, as the space satisfies "Murphy's Law". In genus 1, however, the situation remains beautiful and combinatorial. I will describe a natural smooth compactification of the space of elliptic curves in projective space.

This space is a blow up of the space of stable maps. It can be interpreted as blowing up the most singular locus first, then the next most singular, and so on, but with a twist—these loci are often entire components of the moduli space. I will give a number of applications in enumerative geometry and Gromov–Witten theory. For example, it has been used by Aleksey Zinger to prove physicists' famous mirror symmetry prediction for genus 1 Gromov–Witten invariants of a quintic threefold.

This is joint work with Zinger.

DAVE WAGNER, University of Waterloo

Negative correlation inequalities for random-cluster models

The (anti-ferromagnetic) q-state Potts model of a graph reduces to the number of proper q-colourings of the graph (when q is a natural number and the temperature is zero). The random-cluster expansion gives an interpretation of this partition function for any $q \geq 0$. When $q \geq 1$, the FKG inequality yields positive correlations among any increasing functions on the state space. (At $q = 1$ all the fundamental events are independent.) In the range $0 \leq q \leq 1$ negative correlations are known to hold in some forms, but not in others, and are conjectured to hold in many more. I will survey the current state of the problem, highlighting recent progress and potential applications.