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Convex bodies, isoperimetric inequality and degree of line bundles

We show how to associate a convex body (i.e., a compact connected convex set in R^n) to a finite dimensional subspace of regular functions L on a (quasi) affine variety X (of $\dim n$) so that the volume of this convex set is responsible for the number of solutions of a generic system of equations $\{f_i = 0\}$ in L.

Same construction works to associated a convex body to an ample line bundle on a projective variety. The volume of the convex body is then responsible for the degree of the line bundle. This can be regarded as a generalization of the Newton polytopes and the well-known Kushnirenko theorem in toric geometry as well as the Gelfand–Cetlin polytopes in representation theory. This will have several interesting applications, e.g. the well-known Brunn–Minkowskii inequality (generalization of isoperimetric inequality) in convex geometry gives Hodge index theorem and log-concavity of the degree function of line bundles.