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Projective Hulls, Projective Linking, and Boundaries of Analytic Varieties

This talk revolves generally on the question: When does a real closed curve γ in a complex manifold X bound a holomorphic surface $\Sigma \subset X$? In a classic paper in 1959 John Wermer gave a beautiful answer for the case $X = \mathbf{C}^n$. He showed that γ bounds a one-dimensional subvariety $\Sigma \subset \mathbf{C}^n$ if and only if the restrictions of complex polynomials to γ are not dense in the continuous functions. In fact, let A denote the closure of this algebra of polynomials in $C(\gamma)$. Then Wermer showed that Σ can be identified with the Gelfand spectrum of A.

In a different direction, H. Alexander and J. Wermer later considered oriented curves $\gamma \subset \mathbf{C}^n$ (not necessarily connected) and asked when does γ form the boundary of a positive holomorphic 1-chain (a positive integral linear combination of 1-dimensional subvarieties). They proved that this happens if and only if γ has non-negative linking with all algebraic subvarieties in $\mathbf{C}^n - \gamma$.

I shall discuss analogues of these results in complex projective space and more generally in projective algebraic manifolds. This will entail the notion of the projective hull of a compact subset of projective space and the concept of projective linking numbers. The results extend to higher dimensional submanifolds $M \subset X$. They lead to an interesting duality result concerning the classes represented by positive holomorphic chains. This duality asserts that under the pairing $H_{2p}(X, M : \mathbf{Q}) \times H_{2n-2p}(X_M\ ; \mathbf{Q}) \to \mathbf{Q}$, the classes in each group represented by positive holomorphic chains are polars of each other.