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On the importance of asking the right research questions: Could Jordan have proved the Jordan–Hölder Theorem?

In 1870 Jordan proved that the composition factors of two composition series of a group are the same. Almost 20 years later Hölder (1889) was able to extend this result by showing that the factor groups, which are quotient groups corresponding to the composition factors, are isomorphic. This result, nowadays called the Jordan–Hölder Theorem, is one of the fundamental theorems in the theory of groups.

The fact that Jordan, who was working in the framework of substitution groups, was able to prove only a part of the Jordan–Hölder Theorem is often used to emphasize the importance and even the necessity of the abstract conception of groups, which was employed by Hölder.

However, as a little-known paper from 1873 reveals, Jordan had all the necessary ingredients to prove the Jordan–Hölder Theorem at his disposal (namely, composition series, quotient groups, and isomorphisms), despite the fact that he was considering only substitution groups and that he did not have an abstract conception of groups. Thus, I argue that the answer to the question posed in the title is "Yes".