Probabilistic Methods in Analysis and Algebra
Méthodes probabilistiques en analyse et algèbre
(Org: **Matthias Neufang** (Carleton) and/et **Balint Virag** (Toronto))

MIKLOS ABERT, University of Chicago, US
Invariants and actions of residually finite groups

We analyze various asymptotic invariants of residually finite groups using representations on rooted trees. As an application, we prove a theorem on hyperbolic 3-manifolds.

RAZVAN ANISCA, Lakehead University, Thunder Bay, Canada
Banach spaces with two non-isomorphic complex structures

In considering isomorphisms of complex Banach spaces, a natural question is whether or not real isomorphic spaces are complex isomorphic. A negative answer was given by Bourgain and Szarek, who exhibited, using probabilistic methods, a real Banach space which admits two non-isomorphic complex structures.

We present a constructive version of the Bourgain–Szarek example, as well as similar explicit constructions in contexts where the random techniques are not suitable (as in the class of weak Hilbert spaces).

MICHAEL ANSHELEVICH, Texas A&M University
Hilbert space representations of quadratic exponential families, classical and free

Measures corresponding to quadratic exponential families can be characterized in a number of ways. For example, the generating function for their orthogonal polynomials has a special form, their Laplace transforms satisfy differential equations, *etc*. To this list we add a representation on a modified symmetric Fock space. For most of the talk, we will concentrate on the non-commutative version of these results, which, while more abstract, are in fact somewhat simpler.

SERBAN T. BELINSCHI, University of Waterloo, 200 University Street West, Waterloo, ON, N2L 1V9
Burgers equation in some matrix models

In works of Alice Guionnet—alone, and together with Ofer Zeitouni—have been established connections between limits of some matrix integrals (integrals of the form $I_N^\beta(A_N,B_N) = \int \exp\{\frac{N\beta}{2}\operatorname{tr}(UA_NU^*B_N)\}\,dm_N^\beta(U)$, where m_N^β denotes the Haar measure on the orthogonal group OA_N when $\beta=1$ and on the unitary group $\mathcal{U}_N$ when $\beta=2$, and A_N, B_N are diagonal real matrices), and the Burgers equation. Roughly speaking, the Burgers equation appears in connection to the rate function of the convergence of such integrals. In this talk we will show how these connections can be reinterpreted in terms of the Beltrami equation and we shall use this interpretation to give a more complete description of the rate function. We will apply this to a concrete example, related to the Ising model.

This is joint work with Alice Guionnet.

BENOIT COLLINS, University of Ottawa / Lyon
Convergence of unitary matrix integrals

We prove the real convergence of unitary matrix integrals with a small potential. We mention a few applications to combinatorics, theoretical physics and free entropy theory.

This is joint work with A. Guionnet and E. Maurel-Segala.

GEORGE ELLIOTT, University of Toronto, Toronto, Ontario, M5S 2E4
What is a classification functor?

An abstract functorial approach to classification will be described, including also an abstract isomorphism theorem, based on an approximate intertwining argument, which is applicable whenever a suitable notion of inner automorphism is present—which is decreed to be killed by the classification functor. (Examples will be given.)

WOJCIECH JAWORSKI, Carleton University
Powers of averages of unitary representations

Given a unitary representation π of a locally compact group G and a probability measure μ on G, let P_μ denote the contraction $P_\mu = \int_G \pi(g)\,\mu(dg)$. If $X_1, X_2, X_3 \dots$ is a sequence of i.i.d. G-valued random variables whose common distribution is μ, then the sequence $\pi(X_n X_{n-1} \dots X_1)^{-1} P_\mu^n$ converges almost surely in the strong operator topology. This result and some of its consequences regarding a more explicit description of the asymptotic behaviour of the powers P_μ^n when n tends to ∞, will be discussed.

TODD KEMP, MIT
Haagerup's Inequality in Free Probability

In 1978, Uffe Haagerup introduced an important inequality in the context of the left-regular representation of a free group. While its original intent was to furnish an important counter-example in the theory of C^*-algebras, Haagerup's inequality quickly found numerous important applications in other fields: geometric group theory, Lie theory, random walks on groups, the Baum–Connes conjecture, and more.

The free group, or rather the von Neumann algebra generated by its left-regular representation, is the natural arena for *free probability*, a field which incorporates operator algebraic, probabilistic, and combinatorial techniques. In this talk, I will discuss a strengthened version of Haagerup's inequality, and a generalization thereof to R-diagonal operators—a wide class of non-normal operators important in free probability.

This is joint work with Roland Speicher.

CLAUS KOESTLER, Carleton University
A noncommutative version of de Finetti's theorem

De Finetti's theorem states that an infinite exchangeable sequence of random variables is conditionally independent. An equivalent characterization of exchangeability was given in 1988 by Kallenberg in terms of spreadability.

I present in my talk a new theorem that transfers these classical results to an operator algebraic setting: An infinite exchangeable/spreadable sequence of noncommutative random variables is conditionally independent over its tail algebra. Here the noncommutative version of independence is provided by Popa's commuting squares as they are well known in subfactor theory. Surprisingly and in contrast to the classical results, the notions of exchangeability and spreadability are no longer equivalent. I will illustrate this new phenomena by deformed tensor shifts on a Jones tower.

JAMIE MINGO, Queen's University
Second Order Cumulants of Products

Voiculescu's theory of free probability gives a universal rule for calculating moments of free random variables given the moments of the individual random variables. Second order freeness does the same for the fluctuations of random variables which are free of second order.

In this talk I will show how to extend a theorem of Krawczyk and Speicher for cumulants of products to the second case.

This is joint work with Edward Tan and Roland Speicher.

ALEXANDRU NICA, Dept. of Pure Mathematics, University of Waterloo
Eta-series and a Boolean Bercovici–Pata bijection for bounded k-tuples

Let D be the set of (non-commutative) distributions of k-tuples of selfadjoint elements in a C^*-probability space. On D one has an operation of free additive convolution; let D' be the set of distributions in D which are infinitely divisible with respect to this operation.

In this talk I will describe a bijection $B\colon D \to D'$, obtained in joint work with Serban Belinschi, and which is a multi-variable version of a bijection studied by Bercovici and Pata in the case when $k = 1$. The bijection B is found by looking in parallel at two "transforms" for non-commutative distributions, the eta-series and the R-transform. We prove a theorem of convergence in moments which parallels the Bercovici–Pata result from the case $k = 1$. On the other hand we prove that, quite surprisingly, B is a homomorphism for the operation of free *multiplicative* convolution. An interpretation of this fact is that eta-series share the nice behaviour which R-transforms were known to have in connection to the multiplication of free k-tuples of non-commutative random variables.

VLADIMIR PESTOV, University of Ottawa, Department of Mathematics and Statistics, 585 King Edward Ave., Ottawa, ON, K1N 6N5
Urysohn metric space and Kirchberg approximation property

The Urysohn universal metric space $\mathbb{U}$ is a remarkable object, which can be described (Vershik) as the completion of the integers equipped with a random (or: generic) metric. In many regards, it is similar to the unit sphere $\mathbb{S}^\infty$ of a separable Hilbert space ℓ^2. There are however some properties long since established for the unit sphere (*e.g.* the distortion property) that remain open for the Urysohn space, and vice versa. We will discuss an example of the latter: Connes' Embedding Conjecture, whose analogue for the Urysohn space has been recently settled.

As a consequence of Kirchberg's work, Connes' Conjecture can be reformulated as follows: every pair of commuting subgroups of the unitary group $U(\ell^2)$ can be approximated with pairs of commuting compact subgroups. In this form, the property (which we call Kirchberg property) makes sense for every topological group admitting a chain of compact subgroups with dense union. Even if such groups are very common among "infinite-dimensional" groups (the infinite symmetric group, the groups of measure and measure class preserving automorphisms, *etc.*), it seems the Kirchberg property has never been verified for *any* concrete example. In a recent joint work with V. V. Uspenskij, we have established the Kirchberg property for the group of isometries of the universal Urysohn metric space $\mathbb{U}$.

BRIAN RIDER, University of Colorado at Boulder
The Riccati map in Random Schroedinger and RMT

We explain the relevance of the classical Riccati map in describing the spectral edge for various random Schroedinger operators in finite volume as well as for certain random matrix ensembles. In the latter case these ideas provide a new characterization of the celebrated Tracy–Widom laws of RMT in terms of the explosion probability of a given one-dimensional diffusion.

This represents joint work with both Balint Virag (Univ. Toronto) and Jose Ramirez (Univ. Costa Rica).

JOSEPH ROSENBLATT, University of Illinois at Urbana–Champaign, Department of Mathematics, Urbana, Illinois 61801
Probabilistic Constructions in Ergodic Theory

The study of weighted ergodic theorems and of special operators in ergodic theory has led to many new insights and advances. While harmonic analysis is often an important tool in this study, probabilistic constructions also play a significant role. Examples of the use of probabilistic constructions in ergodic theory will be described.

MICHAEL RUBINSTEIN, University of Waterloo, 200 University Ave. W, Waterloo, ON, N2L 3G1
Hide and Seek—a naive factoring algorithm

I present a factoring algorithm that factors $N = UV$ provably in $O(N^{1/3+\epsilon})$ time. I also discuss the potential for improving this to a sub-exponential algorithm. Along the way, I consider the distribution of solutions (x, y) to $xy = N \bmod a$.

VOLKER RUNDE, University of Alberta, Edmonton, Alberta
Harmonic operators: a look from the dual side

Let G be a locally compact group, and let μ be a probability measure on G. Then a function $\phi \in L^\infty(G)$ is said to be *μ-harmonic* if $\mu * \phi = \phi$. The μ-harmonic functions do not form a von Neumann subalgebra of $L^\infty(G)$, but can be equipped with a product turning them into a von Neumann algebra in its own right. Dual to this situation, for a continuous, positive definite function σ on G with $\sigma(1) = 1$, A. T.-M. Lau and C.-H. Chu called an element x of the group von Neumann algebra $\mathrm{VN}(G)$ of G, *σ-harmonic* if $\sigma \cdot x = x$. Interestingly, the collection of all σ-harmonic elements is a von Neumann subalgebra of $\mathrm{VN}(G)$.

Recently, W. Jaworski and M. Neufang extended the notion of a harmonic function to that of a harmonic operators. In this talk, which is based on joint work with Neufang, we develop a theory of harmonic operators from the dual perspective, thus extending Lau's and Chu's approach.

DAVID SHERMAN, University of California, Santa Barbara
Some aspects of operator theory in von Neumann algebras

Operator theory studies elements of $B(H)$; what happens when $B(H)$ is replaced with a different von Neumann algebra? Some theorems still work, others do not, and both outcomes can reveal interesting phenomena. In this talk I will discuss results concerning approximate equivalence, essential spectra, and subnormal operators. The springboard is a simple description, in terms of spectral measures, of the norm and strong* closures of the unitary orbit of a normal operator in a von Neumann algebra.

BEN STEINBERG, Carleton University, 1125 Colonel By Drive, Ottawa, Ontario, K1S 5B6
Spectral computations for self-similar groups

Grigorchuk and Żuk used a representation of the lamplighter group as a self-similar group to compute the spectral measure of a simple random walk on this group. They also introduced the notion of the Kesten–von Neumann–Serre (KNS) spectral measure for a self-similar group and gave several examples of the computation of such spectral measures.

In joint work with Kambites and Silva, we characterized when the (KNS) spectral measure coincides with the usual spectral measure; in light of our results and an unpublished result of Abert, this coincidence of measures will occur for any self-similar action of a non-elementary hyperbolic group and for any bireversible automaton group. Using a self-similar representation constructed earlier by the speaker and Silva, we computed the spectral measure for a simple random walk on a wreath product $G \wr \mathbb{Z}$, where G is any finite group. This same result was obtained by Dicks and Schick via a different method.

BALAZS SZEGEDY, University Of Toronto, 40 St. George St., Toronto, ON

A generalization of the moment problem

According to a classical theorem, the sequence $a_0, a_1, a_2, \ldots$ is the moment sequence of a random variable if and only if the infinite matrix $A_{i,j} = a_{i+j}$ is positive semi-definite. We study a graph theoretic analogue of this theorem.

Joint work with László Lovász.