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Semigroups of matrices; and operators?

I will discuss some results on irreducible semigroups of matrices that Heydar Radjavi and I have recently obtained, and consider which of these results might possibly generalize to semigroups of operators on Hilbert space.

We have established the following: a transitive, closed, homogeneous semigroup of linear transformations on a finite-dimensional space either has zero divisors or is simultaneously similar to a group consisting of scalar multiples of unitary transformations. The proof begins by establishing the result that for each closed homogeneous semi-group with no zero divisors there is a k such that the spectral radius of AB is at most $kr(A)r(B)$ for all A and B in the semigroup. (It is also shown that the spectral radius is not k-submultiplicative on any transitive semigroup of compact operators.)

We have also proven that an irreducible semigroup of complex matrices is, respectively, finite, countable or bounded if there is a non-zero linear functional whose range on the semigroup has the corresponding property. Moreover, an irreducible semigroup is finite, countable or bounded if it has a nonzero ideal with the corresponding property.

I'll discuss as much of the above as time permits.