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Fitzpatrick functions and (cyclically) monotone operators

In 1988, Simon Fitzpatrick defined a new convex function F_A—nowadays called the Fitzpatrick function—associated with a monotone operator A, and similarly a monotone operator G_f associated with a convex function f.

This talk surveys recent joint works with Alex McLaren (Guelph) and Hristo Sendov (Guelph), and with Jon Borwein (Dalhousie) and Shawn Wang (UBC Okanagan).

We consider the Fitzpatrick function of the subdifferential operator of a proper, lower semicontinuous, and convex function. This operator is cyclically monotone. A refinement of the classical Fenchel–Young inequality is derived and conditions for equality are investigated. The results are illustrated by several examples.

We also study the problem, originally posed by Fitzpatrick, of determining when $A = G_{F_A}$. Fitzpatrick proved that this identity is satisfied whenever A is a maximal monotone; however, he also observed that it can hold even in the absence of maximal monotonicity. We propose a new condition sufficient for this identity, formulated in terms of the polarity notions introduced recently by Martinez–Legaz and Svaiter.