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Two models for transitive closure of bipolar weighted digraphs

A bipolar weighted digraph is a digraph together with a weight function and a sign function on the arcs such that the weight of each arc lies in the interval $[0,1]$ and no two parallel arcs have the same sign. Bipolar weighted digraphs are natural models for so-called fuzzy cognitive maps, which are used in science, engineering, and the social sciences to represent a body of knowledge. It has been noted in the literature that a transitive closure of a bipolar weighted digraph contains useful new information for the fuzzy cognitive map it models.

It is natural to define a transitive closure of a bipolar weighted digraph $D = (V, A)$ as a bipolar weighted digraph $D^* = (V, A^*)$ such that an arc (u, v) of sign s is in A^* if and only if D has a directed (u, v) -walk of sign s (where the sign of a directed walk is defined as the product of signs of all its arcs). But what weight should be assigned to (u, v) in D^* ? We propose two models: the bipolar fuzzy digraph model, which has been previously considered in some form in fuzzy systems research, and the new bipolar random digraph model. A bipolar fuzzy digraph consists of two fuzzy relations on the set V (that is, the arc weights are viewed as degrees of membership), and its transitive closure is a combined transitive closure of the two fuzzy relations. In a random digraph model, on the other hand, the arc weights are viewed as probabilities, and the weight of an arc (u, v) of sign s in the transitive closure is defined as the probability of having a directed (u, v) -walk of sign s in D . While a version of Roy–Warshall's algorithm efficiently computes the transitive closure of a bipolar fuzzy digraph, the problem is computationally hard for bipolar random digraphs. However, we describe several approaches that allow computation at least for the types and sizes of fuzzy cognitive maps we have dealt with in practice.