MARCUS PIVATO, Trent University, 2151 East Bank Drive, Peterborough, Ontario K9L 1Z8

Crystallographic Defects in Cellular Automata

Let $A^{\mathbb{Z}^D}$ be the Cantor space of $\mathbb{Z}^D$-indexed configurations in a finite alphabet A, and let σ be the $\mathbb{Z}^D$-action of shifts on $A^{\mathbb{Z}^D}$. A *cellular automaton* is a continuous, σ-commuting self-map Φ of $A^{\mathbb{Z}^D}$, and a Φ-*invariant subshift* is a closed, (Φ, σ)-invariant subset $X \subset A^{\mathbb{Z}^D}$. Suppose $x \in A^{\mathbb{Z}^D}$ is X-admissible everywhere except for a small region we call a *defect*. It is empirically known that such defects persist under iteration of Φ, and propagate like 'particles' which coalesce or annihilate when they collide. We construct algebraic invariants for these defects, which explain their persistence under Φ, and partly explain the outcomes of their collisions. Some invariants are based on the spectrum or cocycle-structure of X; others arise from the higher-dimensional (co)homology/homotopy groups of X, generalizing methods of Conway, Lagarias, Geller, and Propp. We also study the motion of defect particles (in the case $D = 1$), and show that it falls into several regimes, ranging from simple deterministic motion, to random walks, to the emulation of Turing machines or pushdown automata.