**GABOR TARDOS**, Simon Fraser University / Renyi Institute, Budapest
*Extremal theory of geometric graphs*

In this survey talk we consider how many edges a geometric graph with $n$ vertices may have if it does not contain some specific forbidden configuration as a subgraph. Here a geometric graph is a graph whose vertices are points in general position in the plane and whose edges are straight line segments.

The simplest forbidden configuration to consider is a pair of crossing edges: if no pair of edges crosses, we have a planar graph with at most $3n-6$ edges. Agarwal *et al.* considered three pairwise crossing edges and proved the number of edges is still linear if this forbidden configuration is not contained. Recently Eyal Ackerman extended the same result to four pairwise crossing edges. With Ackerman we found the maximal number of edges in a geometric graph not containing three pairwise crossing edges within a small additive constant. It is a challenge to extend the linear bound to the forbidden configuration of five (or more) pairwise crossing edges. At present Valtr's $O(n \log n)$ bound is the best.

Another natural choice for a forbidden configuration is the self-crossing drawings of a small (planar) graph. With Pach, Pinchasi, and Tóth we proved that the maximal number of edges in a geometric graph not containing a self-crossing path of three edges is $\Theta(n \log n)$. For longer self-crossing paths as forbidden patterns the exact order of magnitude is not known, but it is larger than linear (as shown by a randomized pruning procedure) and is $o(n \log n)$. For forbidden self-intersecting cycles of length $4$ we proved an $O(n^{3/2} \log n)$ bound on the number of edges with Adam Marcus, which is almost tight as abstract graphs with $\Omega(n^{3/2})$ edges and no $C_4$ subgraphs exist. This result found many applications in other parts of combinatorial geometry.

Finding the maximal number of edges for other types of forbidden patterns (and tighter estimates for some of the above patterns) raises many interesting research problems.