NICOLAS BURQ, Université Paris Sud-Orsay, France

Non-linear Schrödinger boundary value problems

The purpose of this talk is to present some recent results obtained in collaboration with P. Gérard and N. Tzvetkov (Université Paris Sud-Orsay). On the well posedness of non-linear Schródinger equations on domains (with Dirichlet boundary conditions):

$$i\partial_t u + \Delta u + F(u) = 0, \quad u|_{\partial\Omega} = 0,\ u|_{t=0} = u_0 \in H_0^s(\Omega).$$

I will present two type of results:

1) In the ball, we show that if the non linearity has a gauge invariance (typically $F(u) = |u|^p u$), the problem is not well posed, even for initial data in some Sobolev spaces above the scaling critical index.

2) If the domain is the exterior of a bounded obstacle satisfying a non-trapping condition, we show that the problem is locally well posed for any initial data in $H_0^1(\Omega)$ or $L^2(\Omega)$ (hence globally well posed in the case of defocusing non-linearities) for a large class of non linearities.