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Two Point functions for families of random matrices and non-crossing annular partitions

(joint work with A. Nica)

Let (Ω, P) be a probability space and X be self-adjoint Gaussian random matrix, *i.e.* $X\colon \Omega \to M_n(\mathbf{C})_{s.a.}$ is a matrix valued random variable with independent and normally distributed entires. If we write $X = (f_{i,j})_{i,j=1}^n$ and X is normalized so that $\mathcal{E}(f_{i,j}) = 0$ for all i and j, and $\mathcal{E}\big(\Re(f_{i,j})^2\big) = \mathcal{E}\big(\Im(f_{i,j})^2\big) = 1/(2n)$ for $i \neq j$ and $\mathcal{E}(f_{i,i}^2) = 1/n$, then E. Wigner showed that

$$\mathcal{E}\big(\mathrm{tr}(X^{2p})\big) = c_p + O(1/n^2)$$

where tr is the normalized trace and c_p is the p-th Catalan number.

We shall show that

$$\mathcal{E}\big(\mathrm{tr}(X^p)\,\mathrm{tr}(X^q)\big) - \mathcal{E}\big(\mathrm{tr}(X^p)\big)\mathcal{E}\big(\mathrm{tr}(X^q)\big) = \frac{\alpha_{p,q}}{n^2} + O(1/n^4)$$

where $\alpha_{p,q}$ is the number of non-crossing annular partitions of an annulus with p vertices on the inner circle and q vertices on the outer circle.