Noncommutative Geometry and Applications
Géométrie non commutative et applications

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KONRAD AGUILAR, Pomona College

The Bures metric and the quantum metric on the density space of a C-algebra: the non-unital case*

Building off work of Farenick and Rahaman, we extend the definition of the density space and the Bures metric to the setting of non-unital C*-algebras equipped with a faithful trace and prove that the Bures metric is a metric in this case and that its topology is weaker than the topology induced by the C*-norm. Furthermore, we prove a Heine-Borel type theorem for C*-algebras and the density space, where we prove that for any C*-algebra (unital or non-unital) equipped with a faithful trace, the density space equipped with the Bures metric topology is not compact if and only if the C*-algebra is infinite dimensional. Next, building off work from some of the authors, we extend the definition of the quantum metric on the density space to the non-unital C*-algebra case by introducing the notion of a quantum Lipschitz triple, which form a subclass of quantum locally compact metric spaces of Latrémolière that utilize Rieffel's notion of a quantum metric. Furthermore, we prove that this quantum metric topology is weaker than the topology induced by the C*-norm and finish the article with an analysis of matrix-valued functions on the quantized interval, where the quantum metric topology on the density space is not compact and the quantum metric is not uniformly equivalent to both the Bures metric and the C*-norm induced metric. This is joint work with Karina Behera, Katrine von Bornemann Hjelmborg, Tron Omland, Gregory Wickham, Nicole Wu, and Adam M. Yassine. This work is partially supported by NSF grant DMS-2316892.

JOE BURRIDGE, University of Nottingham

Fuzzy Geometries with an Internal Space

The product of a non-commutative matrix spectral triple with a simple two-dimensional internal space is considered. The inner fluctuations of a vacuum Dirac operator are calculated, using the standard technique of Connes' one-forms. This results in the non-commutative analogue of a gauge field, as expected, and also fluctuations of the spacetime geometry. In addition, the fluctuations include a derivative operator that depends on the particle charge.

SITA GAKKHAR,

Stochastic quantization and the geometry of open quantum systems

In analogy with stochastic quantization of Euclidean QFT, a quantum Langevin equation associated with the spectral action is sought. For Gaussian type test functions, the Evans-Hudson quantum stochastic differential equation provides a candidate. The structure maps for the qSDE are constructed from the noncommutative Laplacian and live on a space over noncommutatuve differential forms similar to the commutative setting. This motivates viewing a canonical open quantum system as a noncommutative Wiener space and relating the noncommutative Laplacian of products of spectral triples to more general Lindbladians.

ASGHAR GHORBANPOUR, Western University

ATUL GOTHE, University of Warsaw / IMPAN

A quantum CW-complex for quantum real projective spaces

A CW-complex is a topological space which can be built up inductively by attaching n-balls B^n along their boundary (n-1)-spheres S^{n-1}. Quantum CW-complexes generalise the classical construction by dualising the pushouts of topological spaces to pullbacks of C*-algebras, and allowing the C*-algebras to be noncommutative. Topological graphs, introduced by Katsura, provide a generalisation of discrete graph algebras and homeomorphism C*-algebras and provide a convenient framework to describe various well-known C*-algebras. Additionally, they have nice correspondence between the algebraic properties of the C*-algebras, and combinatorial and topological properties of the graphs. Using this notion of topological graphs, I'll discuss a quantum CW-complex structure for quantum real projective spaces.

CAMERON KRULEWSKI, Dalhousie University
Twisted Atiyah–Bott–Shapiro Maps and SPTs

The Atiyah–Bott–Shapiro orientation $MTSpin \to KO$ produces a KO-theory Thom class from a spin manifold. We develop a twisted version of this map for spin-(ℓ, k) manifolds: manifolds with a spin structure on their tangent bundle augmented by $\ell + k$-many line bundles. We compute a dual version of this map, which admits a physical interpretation in terms of fermionic symmetry protected topological phases. This talk is based on joint work with Arun Debray and Luuk Stehouwer.

Arun Debray, Cameron Krulewski, Luuk Stehouwer. "Unraveling the Bott spiral." arXiv:2605.00316

JAMES MINGO, Queen's University
Infinitesimal Freeness

A universal rule for computing mixed moments of independent and unitarily invariant random matrices gives, in the large N limit, the rule for free independence. In the forty years since Voiculescu gave this rule many extensions and generalizations have been found.

One extension, found by Belinschi and Shlyakhtenko, is called infinitesimal freeness. This has been shown to model the case where the random matrices have different scales, however there are cases where the model doesn't cover some standard examples, in particular the Gaussian orthogonal ensemble. Guillaume Cébron and I have found the extension of infinitesimal freeness to the case of orthogonal invariance. In addition to covering the orthogonal case, real infinitesimal freeness also turns out to be the the model of freeness needed for the subleading term of finite freeness, which has recently been found by Arizmandi, Perales, and Vázquez Becerrra.

NATHAN PAGLIAROLI, University of Waterloo
Asymmetric phase transitions in random noncommutative geometries

In this talk we will outline study the asymmetric phases of the quartic type $(0,1)$ and $(1,0)$ Dirac ensembles. The focus of this work is on asymmetric solutions to the Schwinger-Dyson and saddle point equations of these models, whose solution spaces prove deeply intricate. Via the Riemann-Hilbert approach, we are able to give explicit formulae for the eigenvalue distributions and free energy of various solutions. Using Hamiltonian Monte Carlos simulations, we are able to reconstruct the phase structure. Lastly, using bootstrapping with positivity, we are able to reconstruct the eigenvalue distribution of these models from their bootstrapped moments. All three methods show excellent agreement for large matrix size. This talk is based on upcoming joint work with Benedek Bukor, Masoud Khalkhali, Samuel Kováčik, Katarína Magdolenová, and Juraj Tekel.

ILYA SHAPIRO, University of Windsor
Annuli stacking and duoidal categories.

When considering Hopf-cyclic cohomology in the braided setting one is led to a natural notion of a monoidal product on the Hochschild homology category of the underlying braided category. It turns out that this product also appears in factorization homology, as a key ingredient in bridging the algebra and topology in the study of 2d TQFTs; the braided category is the algebraic input. Motivated by duality, we will discuss extending this product from the braided to the duoidal setting.

GIORGOS TSIMPERIS, University of Nottingham