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Deferred Cyclotomic Representations: Exact Algebraic Evaluation of Quantum Group Invariants and q-Hypergeometric Series

The computation of topological invariants for 3-manifolds, such as the Turaev-Viro state sums, relies heavily on the representation theory of the quantum group $U_q(sl_2)$. The fundamental building blocks of these invariants (such as the quantum 6j-symbols) are expressed as complex q-hypergeometric series. Evaluating these rational functions poses severe challenges. Standard evaluations rely on dense polynomial or rational formulations, which trigger expression swell in exact symbolic algebra and severe catastrophic cancellation in finite-precision arithmetic.

In this talk, we introduce the Deferred Cyclotomic Representation (DCR) of the q-hypergeometric series, an algebraic framework that resolves these computational bottlenecks. By mapping the multiplicative structure of q-factorials onto a free abelian group generated by irreducible cyclotomic polynomials, the DCR framework translates algebraic multiplication and division into exact sparse integer vector additions. This isolates the combinatorial skeleton of the series from the target evaluation field. We shall demonstrate how DCR enables the extraction of common polynomial divisors, intrinsically resolves root-of-unity poles, and allows the evaluation of generic tensor category identities in exact cyclotomic fields. QRecoupling.jl open-source package is developed for further research and applications.