Homotopy Theory
Théorie de l'homotopie
(Org: **Martin Frankland** (University of Regina), **Cameron Krulewski** (Dalhousie University) and/et **Daniel Teixeira** (Dalhousie University))

MATTHEW ALEXANDER, (Formerly) University of Regina
A (Pr)operad of (Pr)operad algebras

In this talk we present a construction which takes a coloured operad $\mathcal{O}$ and small category $\mathcal{C}$, and produces a corresponding coloured operad $\mathcal{O}^{\mathcal{C}}$, whose algebras are $\mathcal{C}$-shaped diagrams of $\mathcal{O}$-algebras. We then give an analogous construction for properads.

DANIEL ALMEIDA, University of Ottawa
Strictification of models of generalized algebraic theories

Generalized algebraic theories, introduced by Cartmell in the late 1970s, extend the framework of Lawvere theories by allowing the introduction of dependent sorts. This feature makes such theories convenient, for example, for axiomatizing (1-categories of) higher categorical structures in a type-theoretic language. If T is a generalized algebraic theory, two different kinds of models of T have been traditionally considered: a strict one, where contexts are interpreted as iterated families of sets, and a categorical one, which is a model of the underlying finite-limit sketch (or clan) of the syntactic category C(T). These two versions often match, but not always.

We will discuss this phenomenon and indicate how it relates to a more general issue regarding generalized algebraic theories: they can be arranged into a 2-category, but the morphisms that preserve all available structure are too strict to directly encode the intended categorical semantics. This can be studied by constructing a Quillen model structure on the category of generalized algebraic theories whose homotopy bicategory correctly expresses the weaker semantics - that gives, in particular, a strictification result for models of cofibrant theories.

THEOFANIS CHATZIDIAMANTIS-CHRISTOFORIDIS, University of Western Ontario
Fixed Point Properties in Homotopy Type Theory

Homotopy type theory is a formal system for homotopy-coherent mathematics which can be interpreted in any $(\infty, 1)$-topos. We investigate what it means for a space to have the property "every self-map has a fixed point" in this system, inspired by work of Szymik in the category of topological spaces. After constructing counterexamples, we show that classifying spaces of non-Abelian finite simple groups satisfy this property. Along the way, we compute the homotopy groups of the space of maps into a classifying space in homotopy type theory. Our results are formalised in the Rocq proof assistant.

This talk is based on joint work with Dan Christensen.

TAO GONG, Western University
Toric varieties modulo reflections

Let W be a finite group generated by reflections of a lattice M. If a lattice polytope $P \subset M \otimes_{\mathbb{Z}} \mathbb{R}$ is preserved by W, then the quotient P/W admits an embedding into P as a rational polytope. We show that the quotient of the projective toric variety X_P by W is isomorphic to the toric variety $X_{P/W}$. This answers a question of Horiguchi–Masuda–Shareshian–Song. We also study quotients of real toric varieties, proving that $X_P^{\mathbb{R}}/W$ is contractible when P is a permutohedron. This is joint work with Colin Crowley and Connor Simpson.

SHASHEN GOUNDEN, University of Regina
Steenrod's cohomology realization problem for some monomial ideal rings

If A^* is a graded algebra a classical problem in Algebraic Topology asks if there exits a space Z, such that $H^*(Z) \cong A^*$. We investigate this realisation problem for monomial quotient rings A^* of the form $\mathbb{Z}[x_1, \ldots, x_m] \otimes \Lambda_{\mathbb{Z}}[y_1, \ldots, y_n]$ modulo an ideal generated by monic monomials. We interpret such rings as generalised Stanley–Reisner rings associated to multicomplexes. Within this framework we show that if the even generators of A^* have at most degree 4 then A^* is realizable as the cohomology ring of a homotopy colimit of products of $\mathbb{CP}^i$, $\mathbb{HP}^j$ $0 \leq i, j \leq \infty$ and S^{2k-1} $(k \in \mathbb{N})$ indexed by a sub-poset of the multicomplex.

YANG HU, University of Regina
Realization of some Stanley-Reisner algebras and graph colorings

It is a classical problem in algebraic topology to decide whether a given graded $\mathbb{Z}$-algebra can be realized as the integral cohomology ring of a space. In this talk, we introduce families of Stanley-Reisner algebras depending on graphs and relate their realizability to the span coloring of the graph. This work is joint with Donald Stanley.

DORETTE PRONK, Dalhousie University
Examples of Orthogonal Factorization Systems for Double Categories

Double categories enable us to consider two types of arrows between objects as part of one structure. For instance, ring homomorphisms and modules, or ordinary arrows and spans of arrows. Typically, one types of arrows, the *tight* arrows, composes with strict identities and associativity, whereas the other type of arrows, the *loose* arrows, composes with weak identities and associativity. The connection between the two structures is given through double cells, which come with a tight and loose composition.

In this talk I will introduce orthogonal factorization systems for double categories as pairs of suitably compatible factorization systems on the category of tight arrows and on the category of double cells, with loose arrows as objects and tight composition. The focus of the talk will be on examples of such factorization systems and ways to construct double orthogonal factorization systems that extend a given orthogonal factorization system on the tight arrows of the double category.

Specifically, we will consider the double categories of paths in a 2-category, the double category of spans in a category with pullbacks, the double category of relations (on sets) and if time permits the double category of rings with ring homomorphisms and modules.

DANIEL RAMRAS, Indiana University Indianapolis
Homotopy Pullbacks in CAT

In his thesis, Thomason showed that the Grothendieck construction, applied to a diagram of (small) categories, models the homotopy colimit of the diagram. That is, the nerve of the Grothendieck construction is the homotopy colimit of the diagram of nerves. The dual problem, of finding category-theoretical models for homotopy limits of diagrams in CAT, is much less studied. Quillen's Theorem B and Barwick-Kan's Theorem B_n describe homotopy fibers and homotopy pullbacks categorically, but these models are base-point dependent and give the correct homotopy type only when they happen to be invariant under base-change. Shortly after Quillen's work, M. Evrard constructed a category modeling the homotopy fiber of an arbitrary functor by using zig-zags of morphisms, of indeterminate length, to model paths.

I'll explain a new approach to Evrard's result, which naturally extends to give a zig-zag model for arbitrary homotopy pullbacks in CAT. This leads to a characterization of homotopy cartesian squares in CAT, as well as a purely combinatorial description for the associated Mayer-Vietoris sequence in homotopy (and, in particular, its connecting homomorphism), answering a question arising in work of Roberts et al. on Algebraic Quantum Field Theory.

CARLOS GABRIEL VALENZUELA RUIZ, University of Regina
About tight moment-angle complexes

Let K be a simplicial complex. If the inclusion of any full subcomplex $L \subseteq K$ always induces an injective map in homology, we say K is tight. The property of tightness has been studied before in the context of minimality of manifold triangulations. In this talk, we'll discuss cohomological properties of moment-angle complexes corresponding to tight triangulations of manifolds. This is joint work with Donald Stanley and Daisuke Kishimoto.

DENI SALJA, Dalhousie University

LAURA SCULL, Fort Lewis College
Twisted Bredon Cohomology is a Morita Invariant

Bredon cohomology is an equivariant cohomology theory that is designed to consider information from various fixed sets of the action. In its standard form, it is NOT Morita invariant without restrictions on the coefficients, as shown in [PS]. In order to rectify this, we instead use a a twisted version of Bredon cohomology defined in [MM], indexed over the tom Dieck fundamental groupoid. We prove that this twisted Bredon cohomology is Morita invariant. In this talk, I will explain this result and the concepts that underly it via examples.

This talk is based on joint work with C. Farsi and J. Watts.

[MM] Amiya Mukherjee and Goutam Mukherjee. "Bredon-Illman cohomology with local coefficients". Quart. J. Math. Oxford Ser. (2) 47.186 pp. 199–21 (1996).

[PS] Dorette Pronk and Laura Scull. "Translation Groupoids and Orbifold Bredon Cohomology". Canad. Jour. Math. vol 62 pp 614–645 (2010).

THOMAS WILSKOW THORBJØRNSEN, University of Western Ontario
Finitely Adequate Modules in Synthetic Algebraic Geometry

Synthetic algebraic geometry (SAG) is an extension of homotopy type theory that provides a language for internal reasoning about the big Zariski topos. In SAG, we postulate the existence of a generic local ring R with some additional properties. Schemes over R are not defined by giving the underlying space a structure sheaf; rather, they are defined by a property of the space itself. Sheaves on a scheme are then expressed as bundles over the scheme, and we have many of the usual operations on the sheaves themselves, such as taking cohomology.

However, algebraic geometry often looks different from this internal point of view, compared to the classical external one. For instance, we can show that the generic local ring R is not Noetherian, and so the category of finitely presented R-modules is not abelian. In particular, the cohomology groups of sheaves of finitely presented R-modules may no longer be finitely presented. In this talk, we shall study the abelian closure of the finitely presented R-modules in the category of all R-modules, which we call the finitely adequate R-modules. We will characterize the finitely adequate R-modules which are injective and projective in this subcategory. Then, we prove that finitely adequate R-modules are closed under extensions. We hope that the category of finitely adequate R-modules gives us a suitable replacement for the category of finitely presented modules, so that the cohomology groups of finitely adequate sheaves are finitely adequate.