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Finitely Adequate Modules in Synthetic Algebraic Geometry

Synthetic algebraic geometry (SAG) is an extension of homotopy type theory that provides a language for internal reasoning about the big Zariski topos. In SAG, we postulate the existence of a generic local ring R with some additional properties. Schemes over R are not defined by giving the underlying space a structure sheaf; rather, they are defined by a property of the space itself. Sheaves on a scheme are then expressed as bundles over the scheme, and we have many of the usual operations on the sheaves themselves, such as taking cohomology.

However, algebraic geometry often looks different from this internal point of view, compared to the classical external one. For instance, we can show that the generic local ring R is not Noetherian, and so the category of finitely presented R -modules is not abelian. In particular, the cohomology groups of sheaves of finitely presented R -modules may no longer be finitely presented. In this talk, we shall study the abelian closure of the finitely presented R -modules in the category of all R -modules, which we call the finitely adequate R -modules. We will characterize the finitely adequate R -modules which are injective and projective in this subcategory. Then, we prove that finitely adequate R -modules are closed under extensions. We hope that the category of finitely adequate R -modules gives us a suitable replacement for the category of finitely presented modules, so that the cohomology groups of finitely adequate sheaves are finitely adequate.