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Homotopy Pullbacks in CAT

In his thesis, Thomason showed that the Grothendieck construction, applied to a diagram of (small) categories, models the homotopy colimit of the diagram. That is, the nerve of the Grothendieck construction is the homotopy colimit of the diagram of nerves. The dual problem, of finding category-theoretical models for homotopy limits of diagrams in CAT, is much less studied. Quillen's Theorem B and Barwick-Kan's Theorem B_n describe homotopy fibers and homotopy pullbacks categorically, but these models are base-point dependent and give the correct homotopy type only when they happen to be invariant under base-change. Shortly after Quillen's work, M. Evrard constructed a category modeling the homotopy fiber of an arbitrary functor by using zig-zags of morphisms, of indeterminate length, to model paths.

I'll explain a new approach to Evrard's result, which naturally extends to give a zig-zag model for arbitrary homotopy pullbacks in CAT. This leads to a characterization of homotopy cartesian squares in CAT, as well as a purely combinatorial description for the associated Mayer-Vietoris sequence in homotopy (and, in particular, its connecting homomorphism), answering a question arising in work of Roberts et al. on Algebraic Quantum Field Theory.