Geometry and Quantization
Géométrie et quantification
(Org: **Adrian Chitan** and/et **Lisa Jeffrey** (University of Toronto))

ABDEL RAHMAN AL-ABDALLAH, Brandon University
Homogeneous Levi-Flat Hypersurfaces in Complex Projective Spaces

A classical problem in complex geometry asks whether closed Levi-flat hypersurfaces exist in complex projective spaces, with the case of $\mathbb{CP}^2$ remaining especially delicate. In this talk, I discuss this problem under a natural symmetry assumption: homogeneity. I will consider Levi-flat hypersurfaces in projective space that arise as homogeneous CR manifolds under Lie group actions by projective transformations. Using the structure theory of homogeneous CR manifolds and the geometry of the Levi foliation, I will show that the leaves must be compact homogeneous complex manifolds, hence flag manifolds. In $\mathbb{CP}^2$, this forces the leaves to be projective curves, and Bézout's theorem gives a contradiction to the disjointness of distinct leaves. This yields strong nonexistence results for homogeneous Levi-flat closed hypersurfaces in complex projective spaces.

THOMAS BAIRD, Memorial University of Newfoundland
Moduli spaces of Higgs bundles over a real curve

The moduli space M of stable Higgs bundles over a smooth complex projective curve admits a well-known hyperkahler metric structure. If the curve is real, then the real locus $M(\mathbb{R}) \subset M$ is a so-called BAA-brane, meaning that it is Lagrangian with respect to half of the hyperkahler structure and complex with respect to the other half.

In this talk, I will explain how to calculate the $\mathbb{Z}_2$-Betti numbers of $M(\mathbb{R})$. The proof uses a motivic formula for M due to Schiffmann, Mellit, and Fedorov-Soibelman-Soibelman.

ROBERT CORNEA, University of Waterloo
Stable Wild Vafa-Witten Bundles on $\mathbb{P}^2$

Vafa-Witten bundles were originally introduced in the study of S-duality in $\mathcal{N} = 4$ supersymmetric Yang-Mills theory. Specifically, on a Kähler surface X, a Vafa-Witten bundle is a pair (E, Φ) where E is a holomorphic vector bundle on X and $\Phi \in H^0(X, \mathrm{End}E \otimes K_X)$ called the *Higgs field*. It is known that when $X = \mathbb{P}^2$, non-trivial stable Vafa-Witten bundles do not exist under the standard definition. They do emerge when considering the "wild" case, where in the definition of the Higgs field, one replaces the canonical bundle $K_{\mathbb{P}^2}$ with any line bundle L on $\mathbb{P}^2$. In this talk, we describe the geometry of moduli spaces of rank-two stable wild Vafa-Witten bundles over $\mathbb{P}^2$.

PETER CROOKS, Utah State University
Algebro-geometric symplectic slice theorems and quantization

I will harness the representation theory of complex reductive groups to present algebro-geometric counterparts of the classical symplectic slice theorems. If time permits, I will outline potential applications to algebro-geometric symplectic implosion and quantization. This represents joint work with Rebecca Goldin and Yiannis Loizides.

MARK HAMILTON, Mount Allison University
Convergence of polarization for Gelfand-Tsetlin systems

One of the earliest and most famous examples of "independence of polarization" in geometric quantization is the Gelfant-Tsetlin integrable system, introduced by Guillemin-Sternberg in 1982. They computed the dimensions of the Kahler and real quantizations by different arguments and showed they were equal.

In this talk I will describe an explicit correspondence between these two quantizations using a "convergence of polarizations" approach, as pioneered by Mourao, Nunes, and collaborators. This is a limiting process in which holomorphic sections (which can be seen as elements of the Kahler quantization) converge to distributional sections (which can be seen as elements of the real quantization). I will give an overview of this procedure for the case of "regular" fibres, and discuss some of the issues involved in its construction. I will also discuss work in progress to finish the story by finally tackling the case of "singular" fibres.

This is joint work with Hiroshi Konno and work in progress with Megumi Harada.

OFFICE HOURS,

JACQUES HURTUBISE, McGill University
Quasi-monopoles

(w. R. Bielawski and S. Cherkis)

It has been understood for a while that the end of the moduli space of charge k SU(2) monopoles in R^3 decomposes into regions where one has a glueing of monopoles of charges $k_1, .., k_s$; i.e a picture of well separated particles. This approximation is rather rough, in that comparing the metrics only gives an approximation to order 1/R, where R is the separation parameter. Any further improvement requires some form of interaction between the particles. We define spaces of quasi-monopoles, with a separate spectral curve for each charge, and an interaction through their intersection divisors. The spaces of these quasi-monopoles are hyperkahler, and approximates the monopole metric to order e^{-cR}. They also have torus actions, which allow a way of finalising Segal-Selby's proof of the Sen conjecture in the coprime case.

DMITRY KOROTKIN, Concordia
Quantization of non-abelian Einstein-Rosen waves

Einstein-Rosen cylindrically symmetric waves, when the metric tensor is assumed to be diagonal, an example of explicitly solvable model where the Einstein equations essentially linearize. Quantization of this linear model (summarized in 1996 Ashtekar-Peirri paper) allows to shed some light on general features of quantum gravity.

There is a natural non-abelian generalization of ER waves where the metric is non-diagonal any more and the equations of motion become non-linear but integrable in the sense of the theory of solitons. Algebraic quantization of this model using the theory of quadratic quantum algebras of Yangian type was completed in 1990's in the works of H.Nicolai, H.Samtleben, M.Niedermeier and the speaker. However, at that time no unitary representations of these algebras were found. More recently, it was discovered by M.Reisenberger that identical algebras naturally appear (both at the Poisson and quantum level) as building blocks of the light-front approach to the full 4d gravity. Motivated by this result, in our recent work with J.Peraza and M.Reisenberger we established an isomorphism between the quadratic algebra appearing in the non-abelian quantum ER waves and the S-matrix quadratic algebra arising in the Gross-Neveu model. This opens the way to construction of unitary representations of quantum ER algebra, analysis of the energy and metric spectrum with the ultimate goal to apply this machinery to the full 4d quantum gravity.

The talk is based on earlier works with H.Nicolai (AEI Golm) and H.Samtleben (ENS Lyon), and recent work with J.Peraza (Perimeter) and M.Reisenberger (Montevideo).

JOSÉ MOURÃO, Instituto Superior Técnico, University of Lisbon
Fourier-Helgason transform as infinite geodesic time limit in quantization

We give a geometric quantization interpretation of the Fourier-Helgason (FH) transform for Riemannian symmetric spaces of noncompact type, $X = G/K$.

First, we show that Lisiecki's horizontal polarization is the infinite time limit of the pushforward of the vertical polarization with respect to the geodesic flow for a G–invariant Riemannian metric.

Then we lift the geodesic flow to an intertwining unitary parallel transport on the quantum bundle that we call quantum geodesic transform (QGT). Finally we show that the QGT has a well defined limit as the geodesic time goes to infinity and that is equivalent to the FH transform.

On work in collaboration with A.C. Ferreira, J. Hilgert and J.P. Nunes

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JOÃO NUNES, Instituto Superior Tecnico - University of Lisbon
Quantization and Hamiltonian flows in imaginary time

I will review the application of Hamiltonian flows in imaginary time to problems in geometric quantization. I will describe applications to symplectic toric manifolds and will also describe how this framework provides a geometric interpretation of the Peter-Weyl theorem.

MARTIN PINSONNAULT, Western University

DANIEL RAMRAS, Indiana University Indianapolis
Spaces of flat connections

Given a manifold M and a homomorphism from $\pi_1(M)$ into a Lie group G, one has an associated flat principal bundle over M. The space of flat connections on this bundle measures the failure of this construction to be an equivalence: more precisely, this space is the homotopy fiber of the map taking a homomorphism to the classifying map for its associated bundle.

In this talk, I'll survey results (some joint with T. Baird, and some due to my student A. Davis) on the homotopy type of spaces of flat connections, contrasting the case of surfaces (where Morse Theory for the Yang-Mills functional allows one to analyze flat connections directly) with the situation for higher-dimensional manifolds. In higher dimensions, rational cohomology of M and information about irreducible representations of $\pi_1 M$ can each be leveraged to produce topological information about flat connections.

Time permitting, I'll discuss some ideas for analyzing these phenomena at higher energy (that is, for connections on which the Yang-Mills functional is non-zero), based on work of Atiyah-Bott and Morrison.

OOD SHABTAI, University of Toronto Mississauga
Localization of quantum systems at Liouville tori

We consider a collection of pairwise commuting quantum observables defined via Berezin-Toeplitz quantization of a closed Kähler manifold, and assume that the Arnold-Liouville theorem applies to their principal symbols. We use the joint eigensections of these observables to define isometric embeddings of the quantum spaces into the space of square integrable functions on a fixed Liouville torus; these embeddings may be viewed as a type of semiclassical localization. We discuss applications for contractions of Lie algebra representations and (time permitting) for pairs of spectral projections of quantum observables.

ALEJANDRO URIBE, University of Michigan
Semi-classical aspects of the quantum Zeno effect

The quantum Zeno effect is an interesting consequence of the collapse of the wave function axiom. It leads to the dynamics of Hamiltonians of the form $\hat{H}_Z = \Pi\hat{H}\Pi$, where $\hat{H}$ is a quantum Hamiltonian driving a free dynamics and Π an orthogonal projection. In this talk I will take Π to be a spectral projector of a quantum Hamiltonian circle action with a Kähler phase space (for example, the span of the harmonic oscillator eigenstates with energy ≤ 1). After discussing some general semi-classical aspects of Zeno Hamiltonians of this form, I will focus on the construction of quasi-modes (in one degree of freedom), which turn out to concentrate not only on the level sets of H intersected with the classically-allowed region corresponding to Π (the unit disk in the example), but also on its boundary. The boundary concentration has strong implications for the propagation of singularities under $\hat{H}_Z$. This is joint work with L. Charles, Y. Guedes-Bonthonneau, and S. Vu Ngoc.

JACQUES VAN WYK, University of Waterloo
Generalised Complex Structures on Products of Lie Groups

Let M be an even-dimensional manifold, and let H be a closed three-form on M. An H-twisted generalised complex structure on M is an endomorphism of $TM \oplus T^*M$ which squares to -1, preserves the natural pseudometric of $TM \oplus T^*M$, and whose i-eigenbundle is closed under the H-twisted Dorfman bracket. A natural question is given a fixed closed three-form H on M, does there exist an H-twisted generalised complex structure on M? We explore this question for products of simple Lie groups. This is motivated by Marco Gualtieri's result that any even-dimensional semisimple Lie group admits a generalised complex structure.

JONATHAN WEITSMAN, Northeastern University
Hilbert Polynomials of Calabi Yau Hypersurfaces in Toric Varieties and Lattice Points in Polytope Boundaries

We show that the Hilbert polynomial of a Calabi-Yau hypersurface Z in a smooth toric variety M associated to a convex polytope Δ is given by a lattice point count in the polytope boundary $\partial\Delta$, just as the Hilbert polynomial of M is known to be given by a lattice point count in the convex polytope Δ. Our main tool is a computation of the Euler class in K-theory of the normal line bundle to the hypersurface Z, in terms of the Euler classes of the divisors corresponding to the facets of the moment polytope. We observe a remarkable parallel between our expression for the Euler class and the inclusion-exclusion principle in combinatorics. To obtain our result we combine these facts with the known relation between lattice point counts in the facets of Δ and the Hilbert polynomials of the smooth toric varieties corresponding to these facets.