
ABDEL RAHMAN AL-ABDALLAH, Brandon University
Homogeneous Levi-Flat Hypersurfaces in Complex Projective Spaces

A classical problem in complex geometry asks whether closed Levi-flat hypersurfaces exist in complex projective spaces, with the case of $\mathbb{C}\mathbb{P}^2$ remaining especially delicate. In this talk, I discuss this problem under a natural symmetry assumption: homogeneity. I will consider Levi-flat hypersurfaces in projective space that arise as homogeneous CR manifolds under Lie group actions by projective transformations. Using the structure theory of homogeneous CR manifolds and the geometry of the Levi foliation, I will show that the leaves must be compact homogeneous complex manifolds, hence flag manifolds. In $\mathbb{C}\mathbb{P}^2$, this forces the leaves to be projective curves, and Bézout's theorem gives a contradiction to the disjointness of distinct leaves. This yields strong nonexistence results for homogeneous Levi-flat closed hypersurfaces in complex projective spaces.