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Oppositizing Cells in Equipments

Given an arbitrary double category $\mathbb{D}$, one can define the double category $\mathbb{D}^{con}$ with the same underlying structure as $\mathbb{D}$, but the directions of the cells formally reversed. For equipments (sometimes called framed bicategories or fibrant double categories), that is, double categories for which every tight arrow f has a representative loose arrow (called a companion of f) with a loose right adjoint (called a conjoint of f), the situation becomes more subtle. It is well-known that, in an equipment, an arbitrary cell can be equivalently represented (as a globular cell) in four different ways. One representation requires only the presence of companions, another only conjoints, and the remaining two require both. However, taking opposites of cells is not well-defined on these equivalence classes. Moreover, for the first two types of these representatives, opposites of each of these kinds of cells naturally fit back together into double categories (these are the double categories of retrocells and coretrocells of Paré), while the other two do not in general.

In this talk, we will examine this idea of oppositizing cells through the lens of the formal theory of fibrations, give some conditions for when each of the four kinds of opposite cells fit back together into double categories, and, time permitting, speak on connections to logic and how we believe this connects two different versions of "double categories of relations".