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*Linear and Cyclic *-Autonomous Proarrow Equipments*

There is often more than one type of arrow between objects. Frequently, there are maps which strictly preserve structure and more relaxed morphisms which behave like objects themselves, e.g. functions versus relations. A paradigm to study both simultaneously is given by double categories, introduced by Ehresmann, and pseudo double categories, defined by Grandis and Paré. In particular, these double categories often arise from proarrow equipments, as developed by Wood.

On the other hand, with the advent of linear logic by Girard, and the introduction of cyclic $*$ -autonomous and linear bicategories by Cockett, Koslowski and Seely, it has become clear that certain bicategories have two linked compositions: tensor and par. Indeed, besides standard relational composition, the bicategory Rel is equipped with a relational sum composition. Further examples were then developed by Blute, K-B and Niefield. Crucially, the tensor structures of most examples are canonically proarrow equipments, inducing well-studied double categories. It is then natural to ask how these stricter maps interact with par.

In this talk, we will discuss joint work with Robert Morissette and Dorette Pronk exploring how a proarrow equipment can be compatible with the par of linear bicategories and the linear negation of cyclic $*$ -autonomous bicategories. This compatibility induces new double categories with par as loose composition. We will demonstrate how most examples fit within this paradigm, moreover how piercean bicategories are captured by this new framework, as introduced by Bonchi, Di Giorgio, Trotta to study the cartesian structure of the linear bicategory Rel .