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Complete Equational Theories for Quantum circuits

This talk synthesises recent work on complete sets of relations for quantum circuits. We introduce the first complete equational theory for quantum circuits, thereby answering a question which had been open for two decades. A complete equational theory is a set of equations between circuits that enables one to transform any two equivalent circuits into one another, by successive local rewriting. We obtain this result by means of a back-and-forth encoding between quantum circuits and a graphical language for linear optical circuits, for which we had previously obtained a simple complete equational theory. We show that the set of equations given by this method can be simplified into one made of a few simple and intuitive equations, which we prove to be minimal, in the sense that none of the equations can be derived from the others. One of the equations, although trivial regarding its semantics, acts on an arbitrary number of qubits, and we show that this cannot be avoided in the framework of ancilla-free quantum circuits. Finally, we extend the completeness result to circuits with ancillae and/or qubit discarding, and we show that in this framework, the equation on an arbitrary number of qubits can be removed, leading to an equational theory made of equations acting on at most 3 qubits. Additionally, we also obtain a completeness result for a finitely generated, universal fragment of quantum circuits. This is joint work with Noé Delorme, Nicolas Heurtel, Shane Mansfield, Simon Perdrix, Benoît Valiron, and Renaud Vilmart.