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Notions of Smallness and Completeness for V-Graded Categories

Categories graded by a monoidal category $\mathcal{V}$, which we call $\mathcal{V}$-graded categories, generalize both $\mathcal{V}$-enriched categories and $\mathcal{V}$-actegories. In previous work, we studied a notion of weighted limits for $\mathcal{V}$-graded categories that specializes to recover the usual notion of $\mathcal{V}$-enriched weighted limit. These $\mathcal{V}$-graded weighted limits are equivalently $[\mathcal{V}^{op}, SET]$-enriched weighted limits with representable-valued weights.

In this talk, we continue to explore $\mathcal{V}$-graded weighted limits by first defining a notion of smallness in the $\mathcal{V}$-graded context then identifying equivalent conditions for the existence of all small $\mathcal{V}$-graded weighted limits. Given that $\mathcal{V}$-graded categories are $[\mathcal{V}^{op}, SET]$-categories and that $[\mathcal{V}^{op}, SET]$ is not locally small, there are some complications with the usual notion of smallness. We therefore define our own notions of smallness and local smallness of $\mathcal{V}$-graded categories by requiring that the hom-presheaves be small presheaves. A $\mathcal{V}$-graded weighted limit is small if the shape of its diagram is a small $\mathcal{V}$-graded category. This $\mathcal{V}$-graded notion of smallness specializes to recover the familiar notion of smallness for $\mathcal{V}$-enriched categories. It is well known that a $\mathcal{V}$-category has all small weighted limits if it has all powers and small conical limits. We extend this result to provide equivalent conditions for a $\mathcal{V}$-graded category to admit all small $\mathcal{V}$-graded weighted limits. This talk is based on joint work with Richard Blute and Rory Lucyshyn-Wright.