

---

**Combinatorial Game Theory**  
**Théorie combinatoire des jeux**

(Org: **Svenja Huntemann** (Mount Saint Vincent), **Neil McKay** (UNB Saint John) and/et **Peter Selinger** (Dalhousie University))

---

---

**ALEX CLOW**, Simon Fraser University  
*How Expressive is Digraph Placement?*

Digraph Placement is a normal play ruleset which generalizes many long studied normal play games. It is also the only placement game which is known to be universal, that is for all games  $X$ , there is a Digraph Placement game  $G$  such that  $G = X$ . We will discuss the proof of this universality and explore questions of the form: what is the smallest class of Digraph Placement game needed to express all games? In particular, we will discuss a conjecture by the author that for all non-negative integers  $d$ , there exists number  $f(d)$  such that for all Digraph Placement games  $G$  with out-degree at most  $d$ , the temperature of  $G$  is at most  $f(d)$ . Note that if the claim is true for  $d = 6$ , this would imply Domineering has bounded temperature, verifying a weak version of a longstanding conjecture of Berlekamp. This is joint work with Neil McKay and Alfie Davies.

---

**JEREMIAH HOCKADAY**, Dalhousie University  
*Evaluating positions in Reverse Hex*

Rex, short for Reverse Hex, is a set coloring game in which players try to avoid connecting terminals of their color. Until recently, both Rex and Hex were not examined through the lens of CGT. In this presentation, we develop methods for analyzing Rex positions. We explore how to tell if one position is preferable to another, how to simplify positions, and some special properties of Rex (and antimonotone set coloring games in general).

---

**SVENJA HUNTEMANN**, Mount Saint Vincent University  
*Partizan ArcKayles has unbounded temperature*

Partizan ArcKayles is a game played on a simple graph in which each edge has been coloured blue or red. The two players take turn selecting an edge and removing it and all incident vertices and edges from the graph, with Left only being allowed to select blue edges and Right red edges. We will show that the temperature of Partizan ArcKayles, a measure of how urgent it is for a player to move first in a component, is unbounded even when restricted to trees. We will also discuss some potential new avenues for showing that the temperature in the game Domineering, of which Partizan ArcKayles is a generalization, is bounded above by 2, as has been long conjectured.

This is joint work with Neil McKay and Craig Tennenhouse.

---

**VERONIKA KERAS**, Dalhousie University  
*The Combinatorial Game Theory of Rex+*

In this talk we will present the combinatorial game theory of the game Rex+. Rex+ is a variant of the game Hex, played on a four sided board made out of hexagons. Both players take turns placing as many stones of their colour as they would like on the board, with the objective being to force the other player to connect their two sides. We describe a new ordering, and present some preliminary results on it.

---

**TOMASZ MACIOSOWSKI**, Memorial University of Newfoundland  
*Exploring Invertible Subgroups of Dead-ending and Blocking Universes*

Misère play exhibits less structure than normal play, for example, in unrestricted misère, there are no invertible elements. Restricting the comparison test to universes lets us recover some structure, such as a computable comparison test or a

characterization of the invertible elements. We investigate the invertible subgroups, which are the  $\mathcal{P}$ -free elements, of the dead-ending and blocking universes and discuss challenges caused by losing the other direction of the maintenance/proviso test due to no longer being in a full universe.

---

**NEIL MCKAY**, UNB Saint John

---

**ALEX MEADOWS**, ST. MARY'S COLLEGE OF MARYLAND

*Joy and Complexity of Blokus Games*

Blokus is a delightful 25-year old board game played by 2-4 people, alternately tiling a grid with polyomino pieces. The game is popular and enjoyable in part due to the large range of game play in spite of very simple rules and a hard limit on the number of moves. We claim that it may also be interesting due to its computational complexity. We introduce some 1 and 2 player Blokus variants, including versions of the classic combinatorial games Domineering and Cram, and discuss the relation to classic tiling problems. We show that a couple Blokus puzzle games (for 1 player) are NP-Complete by reduction, and that one 2 player game (with many tiles of a single shape) is PSPACE-Complete, by an argument similar to the recent proof for Battle Sheep. This is joint work with Ian Shehadeh.

---

**ETHAN SAUNDERS**, University of Calgary

*How high can you go? Finding transfinite ordinals in infinite Capture go go*

Capture Go has the same mechanics as Go except the first player to capture a group wins. On a finite go board, we can have "win in  $k$ " puzzles. Positions where Black is winning but where White can prolong the game for at most  $k$  moves. On an infinite board, even more types of puzzle are possible. We can imagine the possibility of a "win in omega" puzzle in which Black is winning but White can choose an arbitrarily high finite number and prolong the game for as many moves. If we have a position in infinite Capture Go which Black will win in finite time, we can assign it an ordinal number. The supremum of all ordinal numbers that correspond to infinite Capture Go positions is called the omegaone of Capture Go. I will introduce the problem of finding the omegaone of Capture Go and present some progress that we have made. This is joint work with Isobel Shaw.

---

**PETER SELINGER**, Dalhousie University

*On 3-terminal positions in Hex*

The theory of passable games is a version of combinatorial game theory over partially ordered sets of atoms. It is suitable for modelling monotone set coloring games such as Hex. In this talk, I will apply this theory to obtain results about 3-terminal positions in Hex. I'll introduce a family of positions called superswitches and give applications to the verification of Hex templates. I'll also give a characterization of pivoting templates, and describe how they can be used to derive a new handicap strategy for  $11 \times 11$  Hex. This is joint work with Eric Demer.