
Combinatorial Design Theory
Théorie des plans combinatoires

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Length-Maximal Nonlinear Codes with Given Singleton Defect–Structure and Bounds

For a linear $[n, k, d]_q$ code, the columns of a generator matrix form a projective arc, and the maximum length is governed by the classical maximal-arc bound $n \leq (s + 1)(q + 1) + k - 2$, where s is the Singleton defect. We show that this same bound holds for all $(n, q^k, d)_q$ codes, with no assumption of linearity. Codes attaining the bound, which we call length-maximal, are necessarily symbol-uniform and have a sharply constrained distance spectrum. They also satisfy a divisibility condition on s that mirrors, but is weaker than, the condition forced on linear codes. An equivalent form of the bound yields an improved Singleton-type inequality that recovers and extends a result of Guerrini, Meneghetti, and Sala for binary systematic codes. When the defect is large, the bound tightens in discrete steps. We also identify several conditions under which nonlinear codes satisfy the Griesmer bound, and close with open problems, grounded by the central question: *can genuinely nonlinear length-maximal codes exist for parameters where no linear codes do?*

SIMON BLACKBURN, Royal Holloway, University of London

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One Sequence to Rule them All.

A binary reflected Gray code (BRGC), denoted by $B(n)$, is an ordering of the 2^n binary n -tuples such that any two consecutive tuples differ in exactly one bit. This object can also be generated using a greedy approach (introduced by Williams). It is well-known that the position of the bit that changes at each step in the generation of $B(n)$ is given by the binary ruler sequence (OEIS A001511). This leads to a loopless ($\mathcal{O}(1)$ -time) generation of $B(n)$ (by Bitner, Ehrlich, and Reingold).

Mixed-radix reflected and modular Gray codes are natural extensions of the BRGC, where the base of each position is not necessarily binary. Their respective loopless algorithms can be found as Algorithm H and Exercise 77 in Section 7.2.1.1 of TAOCP Vol. 4A by Knuth. In this talk, we discuss that both of these Gray codes built on a mixed-radix base are guided by the same change sequence, which allows them to be generated in parallel. We show that modular Gray codes can be generated using a greedy cyclic increment approach. Furthermore, we present a new family of modular Gray codes starting from any tuple w that can be constructed by using the same greedy approach to generate the next word that is lexicographically greater than or equal to w . We show that although this order is not a suffix of the full modular Gray code, its change sequence is a suffix of the latter.

Joint work with Lucia Moura, Brett Stevens, and Aaron Williams.

DAVID PIKE, Memorial University of Newfoundland
Edge-connectivity of vertex-transitive hypergraphs

A graph or hypergraph is said to be vertex-transitive if its automorphism group acts transitively upon its vertices. A classic theorem of Mader asserts that every connected vertex-transitive graph is maximally edge-connected. We generalise this result to hypergraphs and show that every connected linear uniform vertex-transitive hypergraph is maximally edge-connected. By using combinatorial designs, we also show that if we relax either the linear or uniform conditions in this generalisation, then we can construct examples of vertex-transitive hypergraphs which are not maximally edge-connected. This is joint work with Andrea Burgess and Robert Luther.

SAROBIDY RAZAFIMAHATRATRA, Carleton University
Using cyclic codes to solve an Erdős-Ko-Rado type problem on permutation groups

Given a finite transitive group $G \leq \text{Sym}(\Omega)$, we say that a subset $\mathcal{F} \subset G$ is an intersecting set if any two elements of \mathcal{F} agree on some elements of Ω . If $|\Omega| = pq$, for some odd primes $q < p$, and $G \leq \text{Sym}(\Omega)$ is transitive, then it was conjectured that any intersecting set of G has size at most the order of a point stabilizer. This conjecture was recently disproved using certain cyclic codes in \mathbb{F}_q^p . In this talk, I will show that the conjecture is essentially true whenever these cyclic codes in \mathbb{F}_q^p do not exist. This is based on joint work with Roghayeh Maleki and Angelot Behajaina.

SHAHRIYAR POURAKBAR SAFFAR, Memorial University

KIANOOSH SKOKRI, University of Ottawa

DOUG STINSON, University of Waterloo
Optimal Equidistant Codes—A Detective Story

An *equidistant code* $E(n, d, m)$ consists of m binary codewords of length n , such that the hamming distance between any two distinct codewords is exactly d . We study the following problem: (1) Given n (the length of the codewords), determine the maximum m such an $E(n, d, m)$ exists for some d . Denote this maximum value of m (which of course is a function of n) by m^* . (2) Determine the maximum d such that an $E(n, d, m^*)$ exists. Denote this maximum value of d (which is also a function of n) by d^* . (3) Give a strong combinatorial characterization (in terms of symmetric BIBDs) of $E(n, d^*, m^*)$.

Such optimal equidistant codes have been studied at least since the 1970's. However, many results have been reproved multiple times, and some early work seems to be not well well known. Also, the most comprehensive solution to the problem seems to be incomplete. In this talk, we review the history of this problem and attempt to give a complete unified treatment of the known results.

SOPHIE TOMLIN, University of Ottawa