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On Uniquely Colourable Biclique Designs

A decomposition of a graph G is a collection of subgraphs of G whose union partitions the edge set of G. An (m_1, m_2)-biclique decomposition $\mathcal{D}$ of a graph G is a decomposition of G where $\mathcal{D}$ consists of complete bipartite graphs with parts of size m_1 and m_2. A decomposition $\mathcal{D}$ of G is said to be k-colourable if we can assign k colours to the vertices of G so that no member of $\mathcal{D}$ is monochromatic. If a decomposition is k-colourable but not $(k-1)$-colourable, it is called k-chromatic. A k-colourable decomposition is uniquely k-colourable if its colouring is unique up to the permutation of colour classes.

The study of colouring decompositions has been explored in various settings, but few results are known for unique colourability even for the most prominent families of decompositions such as balanced incomplete block designs and cycle systems. In 2003, Forbes showed that there exist uniquely 3-colourable Steiner triple systems of order n for all admissible $n \geq 25$. More recently, Darijani and Pike proved the existence of uniquely k-colourable e-star systems (for all $k \geq 2$ and $e \geq 3$) and uniquely 2-colourable P_4 systems. Lastly, Burgess, Pike, and Pourakbar-Saffar constructed a uniquely 2-colourable 4-cycle system of order n for all admissible $n \geq 49$.

In this talk, we mainly focus on the construction of uniquely k-colourable (r, r)-biclique decompositions of complete multipartite graphs for all $r \geq 2$ and $k \geq 2$ except $(r, k) = (2, 2)$. These constructions are our stepping stone to find uniquely k-colourable $2r$-cycle systems.