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One Sequence to Rule them All.

A binary reflected Gray code (BRGC), denoted by $B(n)$, is an ordering of the 2^n binary n-tuples such that any two consecutive tuples differ in exactly one bit. This object can also be generated using a greedy approach (introduced by Williams). It is well-known that the position of the bit that changes at each step in the generation of $B(n)$ is given by the binary ruler sequence (OEIS A001511). This leads to a loopless ($\mathcal{O}(1)$-time) generation of $B(n)$ (by Bitner, Ehrlich, and Reingold).

Mixed-radix reflected and modular Gray codes are natural extensions of the BRGC, where the base of each position is not necessarily binary. Their respective loopless algorithms can be found as Algorithm H and Exercise 77 in Section 7.2.1.1 of TAOCP Vol. 4A by Knuth. In this talk, we discuss that both of these Gray codes built on a mixed-radix base are guided by the same change sequence, which allows them to be generated in parallel. We show that modular Gray codes can be generated using a greedy cyclic increment approach. Furthermore, we present a new family of modular Gray codes starting from any tuple w that can be constructed by using the same greedy approach to generate the next word that is lexicographically greater than or equal to w. We show that although this order is not a suffix of the full modular Gray code, its change sequence is a suffix of the latter.

Joint work with Lucia Moura, Brett Stevens, and Aaron Williams.